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Restoration of supersymmetric Slavnov-Taylor and Ward identities in presence of soft and spontaneous symmetry breaking

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Abstract:

Supersymmetric Slavnov-Taylor and Ward identities are investigated in presence of soft and spontaneous symmetry breaking. We consider an abelian model where soft supersymmetry breaking yields a mass splitting between electron and selectron and triggers spontaneous symmetry breaking, and we derive corresponding identities that relate the electron and selectron masses with the Yukawa coupling. We demonstrate that the identities are valid in dimensional reduction and invalid in dimensional regularization and compute the necessary symmetry-restoring counterterms.

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1 Introduction

In the past years, great progress has been made in the calculation of quantum corrections to precision observables. The calculation of such corrections necessitates the use of a regularization method in the intermediate steps of the renormalization procedure. Regularization schemes that preserve all or at least most symmetries of the underlying theory are most convenient. However, as far as supersymmetric gauge theories are concerned, a regularization method that respects all symmetries and is mathematically consistent is not yet known: Dimensional regularization (DREG) [1] breaks supersymmetry, while dimensional reduction (DRED) [2], a variation of DREG widely used in practical calculations, is inconsistent [3] and thus cannot work at all orders.

For quantization the basic symmetries of the theory are formulated as relations between renormalized Green functions. In order to test whether the symmetries are respected by the renormalization scheme or not, the corresponding symmetry identities have to be evaluated by an explicit calculation. If symmetry violations occur, the symmetry breakings have to be absorbed by so-called symmetry-restoring counterterms, a procedure often called algebraic renormalization (see Ref. [4] for an introduction).

Supersymmetric gauge theories exhibit a few peculiarities that have to be addressed in the following.

In practice, calculations are almost exclusively carried out in the Wess-Zumino gauge. This yields a minimal number of unphysical particles but makes it necessary to include the supersymmetry algebra in the BRS transformations. As a result, the supersymmetry transformations become non linear, and the invariance of the action under supersymmetry transformations can no longer be expressed by Ward identities [5] but by more complicated Slavnov-Taylor identities [6,7] (see also [8] for a recent discussion).

An additional complication arises from the fact that supersymmetry is softly broken in all phenomenologically relevant supersymmetric models to account for the mass splittings within supermultiplets. Therefore, it is necessary to include the description of soft supersymmetry breaking in the defining symmetry identities of the theory. This has been worked out in Refs. [9,10,12–14]. Furthermore, soft supersymmetry breaking triggers the spontaneous breaking of internal symmetries. Both soft and spontaneous symmetry breaking generate mass terms in the Lagrangian.

Supersymmetric Slavnov-Taylor identities have been formulated for many physically relevant models: For generic supersymmetric Yang-Mills theories in Refs. [7,10,12], for supersymmetric QED (SQED) in Ref. [15], and the Minimal Supersymmetric Standard Model (MSSM) in Ref. [13].

However, the possible violations of these identities in DREG or DRED and the necessary symmetry-restoring counterterms have only been derived in very special cases: In Ref. [11] several supersymmetric Ward identities for self energies have been shown to be fulfilled in DRED, supersymmetric Slavnov-Taylor identities have been investigated for SQED in Ref. [15] and for SQCD with soft breaking in Ref. [16]. In Ref. [16] the symmetry-restoring counterterms for all gauge and gaugino interactions have been derived; thereby, it turned out that the presence of soft supersymmetry breaking terms have no influence on the determination of the counterterms. Up to now, no explicit calculation

of a supersymmetric Slavnov-Taylor identities has been performed for cases where soft supersymmetry breaking is directly relevant, e.g. sparticle masses.

The aim of the present work is to present such a calculation and to investigate the effect of soft supersymmetry breaking on the possible violations of the Slavnov-Taylor identity in DREG and DRED. Because of the complexity of the MSSM we choose a simplified model where the important features of the MSSM, i.e. soft supersymmetry breaking and spontaneous breaking of an internal symmetry, are retained.

The outline of this article is as follows: In Section 2 we define the model and present its symmetries, in particular the Slavnov-Taylor identity. Section 3 contains information of general importance for the explicit calculation of symmetry identities. We discuss how symmetry identities are violated in DREG and DRED and study when soft supersymmetry breaking becomes relevant in Slavnov-Taylor identities. In Section 4 we consider two symmetry identities for vertex functions as an example. We present the results of an analytical evaluation at one-loop level for arbitrary (on- or off-shell) momenta of the external particles, derive the violation of the identities and the corresponding symmetry-restoring counterterms. Our conclusions are presented in Section 6.

2 The model and its symmetries

2.1 Particle content and Lagrangian

The MSSM is a supersymmetric gauge theory with soft supersymmetry breaking and spontaneous breaking of an internal symmetry. Our aim is to construct a model that retains these essential features of the MSSM concerning symmetry breaking and is maximally simplified. Hence we restrict ourselves to SQED extended by an Higgs sector, i.e. we reduce the gauge group and the matter content to the ones of SQED (U(1) gauge group; two chiral supermultiplets $\Phi_{1,2}$ with charges $Q_{1,2} = \mp 1$ corresponding to the left- and right-handed electron) extended by an additional uncharged chiral multiplet Φ_3. The field Φ_3 takes the role of a Higgs field and will acquire a vacuum expectation value (VEV). In addition to U(1) gauge invariance, we require invariance under a continuous R-transformation with R-charges $n = 2/3$ for the chiral multiplets $\Phi_{1,2,3}$.

The corresponding supersymmetric Lagrangian reads in superfield notation

$$\mathcal{L}_{\text{susy}} = \int \mathrm{d}^4\theta \sum_{i=1,2,3} \Phi_i^\dagger e^{eQ_iV} \Phi_i + \left[\int \mathrm{d}^2\theta \left(\frac{1}{4} W^\alpha W_\alpha + W(\Phi) \right) + \text{h.c.} \right], \tag{2.1}$$

where V denotes the gauge superfield and W_α is the corresponding field strength superfield. The superpotential $W(\Phi)$ takes the form

$$W(\Phi) = \frac{1}{6} \sum_{ijk} y_{ijk} \Phi_i \Phi_j \Phi_k \tag{2.2}$$

with the totally symmetric parameters y_{ijk}. Because of gauge invariance there are only two independent parameters $y_{123} = y_{213} = y_{132} = \ldots$ and y_{333}; the others are zero. Owing to the specific choice of the R-charges no dimensionful parameters are possible in $\mathcal{L}_{\text{susy}}$.

The components of the gauge superfield V are the photon and photino fields (A^μ, λ_α, $\bar\lambda^{\dot\alpha}$), while the chiral multiplets $\Phi_{1,2,3}$ contain the component fields (ϕ_L, ψ_L^α), (ϕ_R, ψ_R^α), $(H, \tilde H^\alpha)$, respectively. The Weyl spinors are combined into 4-component spinors for the photino, electron, and Higgsino as follows,

$$\tilde\gamma = \begin{pmatrix} -\mathrm{i}\lambda_\alpha \\ \mathrm{i}\bar\lambda^{\dot\alpha} \end{pmatrix}, \quad \Psi = \begin{pmatrix} \psi_{L\alpha} \\ \bar\psi_R^{\dot\alpha} \end{pmatrix}, \quad \hat H = \begin{pmatrix} \tilde H_\alpha \\ \bar{\tilde H}^{\dot\alpha} \end{pmatrix}. \tag{2.3}$$

Soft breaking of supersymmetry is introduced via power-counting renormalizable and supersymmetric couplings to a chiral ("spurion") superfield η of dimension 0 [12,13]:

$$\begin{aligned}\mathcal{L}_{\mathrm{soft}} = &\int \mathrm{d}^4\theta \sum_{i=1,2,3} \tilde M_{ii}^2 \eta^\dagger \eta \Phi_i^\dagger e^{2eQ_iV}\Phi_i \\ &+ \left[\int \mathrm{d}^2\theta \left(\frac{1}{2}\tilde M_\lambda \eta W^\alpha W_\alpha + \sum_{ijk} \tilde A_{ijk}\eta\Phi_i\Phi_j\Phi_k\right) + \mathrm{h.c.}\right]\end{aligned} \tag{2.4}$$

with the soft supersymmetry breaking parameters $\tilde M_{ii}$, $\tilde M_\lambda$, and $\tilde A_{ijk}$. The component fields of η are called $(a, \chi^\alpha, f + f_0)$ where the auxiliary f-component has a constant shift f_0. We define the 4-spinor

$$\Xi = \begin{pmatrix} \chi_\alpha \\ \bar\chi^{\dot\alpha} \end{pmatrix} \tag{2.5}$$

for later use. This η-multiplet is treated as an external field that does not propagate. As long as η transforms as a chiral multiplet, $\mathcal{L}_{\mathrm{soft}}$ is supersymmetric. If η is replaced by the constant shift,

$$\eta \to \theta\theta f_0, \quad (a, \chi, f + f_0) \to (0, 0, f_0), \tag{2.6}$$

$\mathcal{L}_{\mathrm{soft}}$ breaks supersymmetry softly. At the same time, continuous R-invariance is explicitly broken.[1]

Interactions like $\eta W^\alpha W_\alpha$ or $\eta\Phi^3$ are not present in power-counting renormalizable supersymmetric models that contain only dimension = 1 chiral superfields. Such interactions have not been considered so far in checks of symmetry identities at the regularized level. The presence of such spurion interactions makes the study of symmetry identities particularly interesting (see Section 4) since they might lead to additional violations in DREG or even in DRED.

The dimensionful parameters in $\mathcal{L}_{\mathrm{soft}}$ lead to a scalar potential with a minimum at a finite VEV, $\langle H\rangle = v$. As a consequence, R-invariance is spontaneously broken and the scalar and fermionic particles become massive. In the following we split off the VEV v from the Higgs field H by the replacement

$$H \to H + v. \tag{2.7}$$

[1]The remaining discrete symmetry is simply a parity transformation where all spinors have parity -1 and all other fields $+1$. Invariance under this transformation is a trivial consequence of Lorentz invariance and does not correspond to R-parity. Anyway, the model is invariant under R-parity where the Standard-Model like fields (photon, electron, Higgs boson) have parity $+1$ and their superpartners -1.

Soft and spontaneous symmetry breaking lead, as in the MSSM, to a relation between the Yukawa coupling y_{123} for the $H\psi_L\psi_R$- or $\tilde{H}\psi_L\phi_R$ interactions, the electron mass

$$m_e = vy_{123}, \tag{2.8}$$

and the selectron mass matrix

$$M_\phi^2 = \begin{pmatrix} m_e^2 + M_{11}^2 & y_{333}vm_e/2 + 6vA_{123} \\ y_{333}vm_e/2 + 6v\tilde{A}_{123} & m_e^2 + M_{11}^2 \end{pmatrix} \tag{2.9}$$

with $M_{11}^2 = f_0^2\tilde{M}_{11}^2$ and $A_{123} = f_0\tilde{A}_{123}$.

In contrast to the MSSM, there are no gauge bosons in this model that couple to Higgs fields. Therefore, gauge invariance is not broken spontaneously, and there are no massive gauge bosons. A second consequence is that no D-terms $\propto e^2v^2$ contribute to the mass matrix M_ϕ^2 in our model.

2.2 Symmetry identities

In order to complete the definition of our model as a renormalized quantum field theory, we rewrite in the following the symmetry requirements in form of Slavnov-Taylor and Ward identities.

Because of the ambiguity inherent in the regularization and renormalization procedure, specifying the Lagrangian is not sufficient to define the model at higher orders. Instead, the model has to be defined by requiring that Γ, the renormalized effective action or generating functional of one-particle irreducible Green functions, has the same symmetry properties as the classical action Γ_{cl}. For this purpose, the symmetries have to be formulated in terms of well-defined identities for Γ that have to be fulfilled in all orders. The structure of such identities for the case of supersymmetric gauge theories, quantized in the Wess-Zumino gauge, has been studied in Refs. [6,7,10,12,13,15].

Gauge invariance and supersymmetry are treated simultaneously by introducing BRS transformations comprising gauge symmetry, supersymmetry, and translations. This requires three kinds of ghost fields: one Faddeev-Popov ghost field corresponding to the gauge transformations $c(x)$ (a fermionic scalar), a supersymmetry ghost ϵ_α (a bosonic spinor), and a translational ghost ω^μ (a fermionic vector). As supersymmetry is global, neither the supersymmetry ghost nor the translational ghost are dynamical fields. We refer to Appendix B for the explicit form of the BRS transformations.

Generally, if symmetry transformations of the classical action are non linear in propagating fields, not only Γ receives loop corrections but also the field transformations themselves. In our case, the non-linear BRS transformations $s\varphi_i$ (see Appendix B) receive loop corrections and have to be renormalized. This is usually done by coupling them to external sources Y_{φ_i} and including them in the effective action.

The gauge-fixing and ghost terms can be conveniently written as a total BRS variation. For this purpose, the antighost $\bar{c}$ and an auxiliary field B are introduced with suitable BRS variations. The complete form of the effective action in lowest order, the classical action Γ_{cl}, is given in Appendix A.

The classical action satisfies the Slavnov-Taylor identity $\mathcal{S}(\Gamma_{\rm cl}) = 0$ with the Slavnov-Taylor operator (given for Weyl spinors):

$$
\begin{aligned}
\mathcal{S} &= \mathcal{S}_0 + \mathcal{S}_{\rm soft}, \qquad (2.10)\\
\mathcal{S}_0(\mathcal{F}) &= \int \mathrm{d}^4x \, \Bigg(sA^\mu \frac{\delta\mathcal{F}}{\delta A^\mu} + sc\frac{\delta\mathcal{F}}{\delta c} + s\bar{c}\frac{\delta\mathcal{F}}{\delta\bar{c}} + sB\frac{\delta\mathcal{F}}{\delta B} \\
&\quad + \frac{\delta\mathcal{F}}{\delta Y_{\lambda\alpha}}\frac{\delta\mathcal{F}}{\delta\lambda^\alpha} + \frac{\delta\mathcal{F}}{\delta Y_{\bar\lambda}^{\dot\alpha}}\frac{\delta\mathcal{F}}{\delta\bar\lambda_{\dot\alpha}} \\
&\quad + \frac{\delta\mathcal{F}}{\delta Y_{\phi_L}}\frac{\delta\mathcal{F}}{\delta\phi_L} + \frac{\delta\mathcal{F}}{\delta Y_{\phi_L^\dagger}}\frac{\delta\mathcal{F}}{\delta\phi_L^\dagger} + \frac{\delta\mathcal{F}}{\delta Y_{\psi_{L\alpha}}}\frac{\delta\mathcal{F}}{\delta\psi_L^\alpha} + \frac{\delta\mathcal{F}}{\delta Y_{\bar\psi_L}^{\dot\alpha}}\frac{\delta\mathcal{F}}{\delta\bar\psi_{L\dot\alpha}} + (L\to R) \\
&\quad + s(H+v)\frac{\delta\mathcal{F}}{\delta H} + s(H^\dagger+v)\frac{\delta\mathcal{F}}{\delta H^\dagger} + \frac{\delta\mathcal{F}}{\delta Y_{\tilde H}}\frac{\delta\mathcal{F}}{\delta\tilde H} + \frac{\delta\mathcal{F}}{\delta Y_{\bar{\tilde H}}}\frac{\delta\mathcal{F}}{\delta\bar{\tilde H}}\Bigg) \\
&\quad + s\epsilon^\alpha\frac{\partial\mathcal{F}}{\partial\epsilon^\alpha} + s\bar\epsilon_{\dot\alpha}\frac{\partial\mathcal{F}}{\partial\bar\epsilon_{\dot\alpha}} + s\omega^\nu\frac{\partial\mathcal{F}}{\partial\omega^\nu}, \qquad (2.11)\\
\mathcal{S}_{\rm soft}(\mathcal{F}) &= \int \mathrm{d}^4x \, \Bigg(sa\frac{\delta\mathcal{F}}{\delta a} + sa^\dagger\frac{\delta\mathcal{F}}{\delta a^\dagger} + s\chi^\alpha\frac{\delta\mathcal{F}}{\delta\chi^\alpha} + s\bar\chi_{\dot\alpha}\frac{\delta\mathcal{F}}{\delta\bar\chi_{\dot\alpha}} \\
&\quad + s(f+f_0)\frac{\delta\mathcal{F}}{\delta f} + s(f^\dagger+f_0)\frac{\delta\mathcal{F}}{\delta f^\dagger}\Bigg). \qquad (2.12)
\end{aligned}
$$

We abbreviate this by

$$
\mathcal{S}(\mathcal{F}) \equiv \int \mathrm{d}^4x \left(\sum_i s\varphi'_i \frac{\delta\mathcal{F}}{\delta\varphi'_i} + \frac{\delta\mathcal{F}}{\delta Y_{\varphi_i}}\frac{\delta\mathcal{F}}{\delta\varphi_i}\right), \qquad (2.13)
$$

where the fields with linear BRS transformations are denoted by φ'_i and the fields with non-linear BRS transformations are denoted by φ_i. The corresponding linearized Slavnov-Taylor operator reads

$$
s_{\mathcal{F}} = \int \mathrm{d}^4x \left(\sum_i s\varphi'_i \frac{\delta}{\delta\varphi'_i} + \frac{\delta\mathcal{F}}{\delta Y_{\varphi_i}}\frac{\delta}{\delta\varphi_i} + \frac{\delta\mathcal{F}}{\delta\varphi_i}\frac{\delta}{\delta Y_{\varphi_i}}\right). \qquad (2.14)
$$

It satisfies

$$
\mathcal{S}(\mathcal{F}+\delta\mathcal{F}) = \mathcal{S}(\mathcal{F}) + s_{\mathcal{F}}\delta\mathcal{F} + \mathcal{O}(\delta\mathcal{F}^2). \qquad (2.15)
$$

In addition to supersymmetry and gauge invariance, we require invariance under continuous R-transformations with the R-charges 2/3 for $\Phi_{1,2,3}$ and 0 for η. The corresponding Ward operator is given by

$$
\mathcal{W}(\mathcal{F}) = \int \mathrm{d}^4x \, \mathrm{i} \left(\sum_i n_{\varphi'_i}(\varphi'_i + v_i)\frac{\delta\mathcal{F}}{\delta\varphi'_i} + n_{\varphi_i}\varphi_i\frac{\delta\mathcal{F}}{\delta\varphi_i} + n_{Y_{\varphi_i}} Y_{\varphi_i}\frac{\delta\mathcal{F}}{\delta Y_{\varphi_i}}\right). \qquad (2.16)
$$

Here v_i denotes the finite VEV of the field φ'_i and is only non-zero for $\varphi'_i = H, H^\dagger, f, f^\dagger$. The R-charges are listed in Table 2.2. R-symmetry is spontaneously and explicitly broken as can be seen by the appearance of v and f_0 in Eq. (2.16).

To summarize, the model is defined by its field content and the following conditions and symmetry requirements on the effective action Γ:

Fields	A_μ	λ^α	ϕ_L	ϕ_R	ψ_L^α	ψ_R^α	H	$\tilde{H}^\alpha$	ϵ^α	a	χ^α	$\hat{f}$
R-charges	0	1	2/3	2/3	−1/3	−1/3	2/3	−1/3	1	0	−1	−2

Table 1: R-charges of the component fields

- Slavnov-Taylor identity and nilpotency of s_Γ:

$$\mathcal{S}(\Gamma) = 0, \tag{2.17}$$
$$s_\Gamma^2 A^\mu = 0. \tag{2.18}$$

  This identity contains gauge invariance, supersymmetry, and translational invariance. The abelian gauge group makes it necessary to impose an additional condition (2.18) to guarantee the nilpotency of the Slavnov-Taylor operator [13,15].

- Gauge-fixing condition, ghost and anti-ghost equation, and translational ghost equation:

$$\frac{\delta\Gamma}{\delta B} = \frac{\delta\Gamma_{\rm cl}}{\delta B}, \quad \frac{\delta\Gamma}{\delta\bar{c}} = \frac{\delta\Gamma_{\rm cl}}{\delta\bar{c}}, \quad \frac{\delta\Gamma}{\delta c} = \frac{\delta\Gamma_{\rm cl}}{\delta c}, \quad \frac{\delta\Gamma}{\delta\omega^\mu} = \frac{\delta\Gamma_{\rm cl}}{\delta\omega^\mu}. \tag{2.19}$$

  These equations hold since they are linear in propagating fields, and they express the non-renormalization of the gauge-fixing term and the usual QED-Ward identity.

- Ward identity for R-symmetry:

$$\mathcal{W}(\Gamma) = 0. \tag{2.20}$$

- Global symmetries: We require that Γ is Lorentz invariant, bosonic, and electrically neutral. Furthermore, Γ has to be invariant under the discrete symmetries C, CP, and must not possess a ghost charge.

- The physical content of Γ is given in the limit

$$\Gamma_{\rm phys} = \Gamma|_{a=\chi=f=0}. \tag{2.21}$$

  In this limit, supersymmetry is softly broken.

3 Renormalization and symmetry-restoring counterterms

At higher orders, the symmetry identities are generally violated at the regularized level. They have to be restored by adding suitable counterterms, so-called symmetry-restoring counterterms. This section contains useful information on how the symmetries can be broken in dimensional schemes and how the symmetry-restoring counterterms can be calculated. The symmetry identities determining these counterterms are classified using power-counting arguments and R-invariance.

3.1 Structure of counterterms and symmetry-breaking terms

The calculation of Γ at higher orders is an inductive process. If the classical action Γ_{cl} and the counterterms up to order $\hbar^{n-1}$, $\Gamma_{\text{ct}}^{(\leq n-1)}$, are known, the regularized effective action $\Gamma_{\text{reg}}^{(\leq n)}$ can be calculated up to order $\hbar^n$. The renormalized effective action up to this order is obtained by adding the counterterms $\Gamma_{\text{ct}}^{(n)}$:

$$\Gamma^{(\leq n)} = \Gamma_{\text{reg}}^{(\leq n)} + \Gamma_{\text{ct}}^{(n)}. \tag{3.1}$$

The counterterms are necessary to cancel ultra-violet divergences and to restore the symmetry identities. They can be split into a symmetry-restoring part and a part that does not interfere with the symmetries,

$$\Gamma_{\text{ct}}^{(n)} = \Gamma_{\text{ct,restore}}^{(n)} + \Gamma_{\text{ct,sym}}^{(n)}. \tag{3.2}$$

$\Gamma_{\text{ct,sym}}$ contains the usual counterterms corresponding to multiplicative renormalization of the parameters and fields.

We now focus on symmetry violations of Slavnov-Taylor identities induced by the regularization scheme. Assuming that the Slavnov-Taylor identity is valid at the order $\hbar^{n-1}$, $\mathcal{S}(\Gamma^{(\leq n-1)}) = 0$, we have

$$\mathcal{S}(\Gamma_{\text{reg}}^{(\leq n)}) = \Delta^{(n)}. \tag{3.3}$$

If the symmetry is free of anomalies, like in our model, $\Delta^{(n)}$ can be absorbed by symmetry-restoring counterterms satisfying

$$s_{\Gamma_{\text{cl}}} \Gamma_{\text{ct,restore}}^{(n)} = -\Delta^{(n)}, \tag{3.4}$$

$$\mathcal{S}(\Gamma_{\text{reg}}^{(\leq n)} + \Gamma_{\text{ct,restore}}^{(n)}) = 0 + \mathcal{O}(\hbar^{n+1}). \tag{3.5}$$

The symmetric counterterms $\Gamma_{\text{ct,sym}}^{(n)}$ satisfy by definition

$$s_{\Gamma_{\text{cl}}} \Gamma_{\text{ct,sym}}^{(n)} = 0. \tag{3.6}$$

They cancel divergences in $\Gamma_{\text{reg}}^{(\leq n)} + \Gamma_{\text{ct,restore}}^{(n)}$, and their finite parts correspond to the free parameters of the model and are fixed by suitable renormalization conditions. Without loss of generality, we can require that $\Gamma_{\text{ct,restore}}^{(n)}$ does not contribute to those vertex functions of order $\hbar^n$ on which renormalization conditions are imposed. The separation (3.2) is then unambiguous.

In the following, we list several important properties of the symmetry-breaking terms $\Delta^{(n)}$.

1. The Quantum Action Principle [17] requires $\Delta^{(n)}$ to be a local, power-counting renormalizable polynomial in the fields of ghost number +1. Furthermore, using the algebraic nilpotency properties of the operators s_Γ and $\mathcal{S}(\Gamma)$, one can derive the condition

   $$s_{\Gamma_{\text{cl}}} \Delta^{(n)} = 0. \tag{3.7}$$

   These properties are important in algebraic proofs of the absence of anomalies and the renormalizability [6,7,18].

2. In DREG as defined in Refs. [19,20] (HVBM scheme) the Quantum Action Principle can be made more precise ("regularized action principle"). As shown in Ref. [20], the breaking Δ of any given symmetry identity can be directly computed:

$$\Delta = \left[\int \mathrm{d}^D x\, \delta\mathcal{L}_{\mathrm{cl+ct}}\right] \cdot \Gamma, \tag{3.8}$$

where $[\mathcal{O}] \cdot \Gamma$ denotes an insertion of the operator $\mathcal{O}$ and $\int \mathrm{d}^D x\, \delta\mathcal{L}_{\mathrm{cl+ct}}$ is the variation of the classical action and counterterms under the corresponding symmetry transformations in D dimensions. E.g. for the cases of the Slavnov-Taylor identity and R Ward identity it is given by $\mathcal{S}(\Gamma_{\mathrm{cl+ct}})$ and $\mathcal{W}\Gamma_{\mathrm{cl+ct}}$ evaluated in D dimensions. For a symmetry identity that is valid at the classical level, $\delta\mathcal{L}_{\mathrm{cl}}$ vanishes in four dimensions and has the form

$$\delta\mathcal{L}_{\mathrm{cl}} = \mathcal{O}(D-4, \hat{g}^{\mu\nu}), \tag{3.9}$$

where $\hat{g}^{\mu\nu}$ is the $(D-4)$-dimensional part of the metric tensor. There are three important consequences for the one-loop breaking $\Delta^{(1)}$:

- At the one-loop level, Feynman diagrams contain only one insertion of $\delta\mathcal{L}_{\mathrm{cl}}$. Since the divergences of the diagrams are at most of the order of $1/(D-4)$, the breaking term $\Delta^{(1)}$ is finite in the limit $\hat{g}^{\mu\nu} \to 0$, $D \to 4$.
- The variation $\delta\mathcal{L}_{\mathrm{cl}}$ is a polynomial in coupling constants, masses, kinematic variables, and fields. The divergences of one-loop integrals have the same structure. Hence, the breaking terms $\Delta^{(1)}$ are not only polynomials in fields and momenta, but even in mass parameters. Particularly $\Delta^{(1)}$ contains no logarithms of mass parameters.
- Moreover, the Quantum Action Principle becomes rather simple at the one-loop level since counterterms appear only in tree diagrams and do not contribute to loops. In higher orders, the interplay of lower-order counterterms and loop contributions is crucial to ensure that the breaking $\Delta^{(n)}$ is local.

3. The statements of Item 2 are even valid in the naive version of DREG, where an anticommuting γ_5 and accordingly $\mathrm{Tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 0$ are used. Of course, naive DREG is problematic since the limit $D \to 4$ of finite quantities such as $\mathrm{Tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 0$ does not agree with the four-dimensional result, but nevertheless this scheme is useful in practical calculations where such traces do not appear.

   Other practically useful schemes are DRED or DREG where an anticommuting γ_5 is used and $\mathrm{Tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma)$ is set to its four-dimensional value at the same time. However, in these schemes the regularized action principle is not necessarily valid because they are mathematically inconsistent, i.e. one initial expression can lead to several, disagreeing results depending on the order of the calculational steps. The results obtained in such inconsistent schemes are in agreement with all physical requirements and can be used if they differ only by local contributions of dimension ≤ 4 from the results obtained in a consistent scheme.

4. In the HVBM scheme, R-invariance is broken at the regularized level because $\int d^D x\, \delta\mathcal{L}_{\rm cl} = \mathcal{W}\Gamma_{\rm cl} \neq 0$ in D dimensions owing to the non-anticommuting γ_5. In naive DREG, on the other hand, R-invariance is fulfilled on the regularized (one-loop) level since $\int d^D x\, \delta\mathcal{L}_{\rm cl} = \mathcal{W}\Gamma_{\rm cl} = 0$ in D dimensions. If only R-invariant counterterms are added, $\int d^D x\, \delta\mathcal{L}_{\rm cl+ct} = \mathcal{W}\Gamma_{\rm cl+ct} = 0$ and R-invariance is valid without introducing symmetry-restoring counterterms. In both versions of DREG, gauge invariance and Eqs. (2.19) are not violated by the regularization scheme.

   Therefore, when naive DREG is used, the symmetry-restoring counterterms $\Gamma_{\rm ct,restore}$ have to satisfy by themselves all the Eqs. (2.19) and (2.20), in particular they are R-invariant. The breaking of the Slavnov-Taylor identity $\mathcal{S}(\Gamma) = \Delta$ is restricted accordingly, in particular

$$\mathcal{W}\Delta = 0. \tag{3.10}$$

3.2 Calculating the symmetry-restoring counterterms

Our aim in this subsection is to sketch the calculation of the counterterms $\Gamma_{\rm ct,restore}$ that restore the Slavnov-Taylor identity. Lorentz and R-invariance are assumed to be respected by the regularization scheme. Several strategies for such calculations have been proposed in the literature: In Ref. [21], Taylor expansions in the momenta are used to generate universal, regularization-independent counterterms in an intermediate step of the calculation of symmetry restoring counterterms. In Ref. [22], the breaking Δ is computed directly using the regularized action principle (3.8) and the equation $\Delta = -s_{\Gamma_{\rm cl}}\Gamma_{\rm ct,restore}$ is solved explicitly.

The strategy we use has often been applied in the literature (see e.g. Refs. [11,15,16]): Slavnov-Taylor identities of the form

$$0 \stackrel{!}{=} \left.\frac{\delta\mathcal{S}(\Gamma)}{\delta\varphi_{k_1}\ldots\delta\varphi_{k_N}}\right|_{\varphi=0} \tag{3.11}$$

are derived and the appearing Green functions are evaluated in the chosen regularization scheme. Without adding counterterms, the identities are in general violated. To restore these identities, symmetry-restoring counterterms are determined in such a way that all Slavnov-Taylor identities are fulfilled.

Generically, the identity (3.11) is a sum of terms of the form

$$\Gamma_{\mathcal{M}Y_{\varphi_i}}\Gamma_{\tilde{\mathcal{M}}\varphi_i}, \qquad \frac{\delta\int d^4x\, s\varphi'_i}{\delta\mathcal{M}}\Gamma_{\tilde{\mathcal{M}}\varphi'_i} \tag{3.12}$$

with monomials $\mathcal{M}, \tilde{\mathcal{M}}$ satisfying $\mathcal{M}\tilde{\mathcal{M}} = \varphi_{k_1}\ldots\varphi_{k_N}$. When these products are evaluated at the one-loop level, Eq. (3.11) becomes a linear expression in one-loop Feynman integrals and also in one-loop counterterms. The role of the symmetry violations and the counterterms is emphasized by rewriting $\mathcal{S}(\Gamma) = 0$ as

$$s_{\Gamma_{\rm cl}}\Gamma^{(1)}_{\rm ct} = -\mathcal{S}(\Gamma^{(1)}_{\rm reg}) = -\Delta^{(1)}. \tag{3.13}$$

Taking the derivative $\delta/(\delta\varphi_{k_1}\ldots\delta\varphi_{k_N})$, identity (3.11) can be equivalently written as

$$\left.\frac{\delta s_{\Gamma_{\rm cl}}\Gamma^{(1)}_{\rm ct}}{\delta\varphi_{k_1}\ldots\delta\varphi_{k_N}}\right|_{\varphi=0} = -\left.\frac{\delta\Delta^{(1)}}{\delta\varphi_{k_1}\ldots\delta\varphi_{k_N}}\right|_{\varphi=0}. \tag{3.14}$$

The breaking term $\Delta^{(1)}$ can be expanded in a basis of monomials,

$$\Delta^{(1)} = \sum_j a_j \Delta_j. \tag{3.15}$$

Δ_j denotes a monomial, i.e. a product of fields and derivatives with ghost number +1 and dimension ≤ 4 which are Lorentz and R-invariant. Each monomial Δ_j corresponds to one identity of the form (3.14) with the fields $\varphi_{k_1} \dots \varphi_{k_N}$ taken from Δ_j. There are in general several monomials containing the same fields $\varphi_{k_1} \dots \varphi_{k_N}$ but differing e.g. in the number of derivatives. One specific monomial can be extracted out of Eq. (3.14) by taking only those terms from Eq. (3.14) that have the same momentum dependence as Δ_j (after Fourier transform in momentum space).

If all identities corresponding to the monomials Δ_j are satisfied by a suitable choice of counterterms $\Gamma^{(1)}_{\text{ct}}$, the Slavnov-Taylor identity is restored. In this way, the restrictions on the Δ_j imply restrictions on the identities that have to be considered. In the following, we present a classification of the Δ_j and discuss the corresponding identities.

The criteria of this classification are based on power counting and R-invariance. The fields appearing in Δ_j are denoted by $\varphi_{k_1} \dots \varphi_{k_N}$ and the power-counting dimension of $\varphi_{k_1} \dots \varphi_{k_N}$ is called d. We set the power-counting dimensions of the dynamical fields to their space-time dimensions. In general, we know that each monomial is a Lorentz scalar of ghost number +1 that does not depend on the ghosts c, ω^μ, and on the fields B, $\bar{c}$ owing to manifest gauge invariance and the identities (2.19). Hence, Δ_j must contain at least one ϵ_α or $\bar{\epsilon}^{\dot\alpha}$ ghost. In Ref. [12] it has been shown that the Slavnov-Taylor identity can be considered in the limit $a = \chi = 0$ without losing information on Green functions that do not depend on spurion and Y-fields. Along the same lines, it can be shown that in our case, where R-invariance is respected by the regularization scheme, even $f = 0$ can be used [14]. Hence only Δ_j without a, χ, f are needed.

$\dim\Delta_j = 4$: Since R-symmetry relates monomials with non-vanishing R-charge to higher-dimensional ones, but Δ contains only monomials of dimension ≤ 4, Δ_j must have R-charge zero. The number of space-time derivatives in Δ_j is $4 - d$ and, thus, the relevant identity reads

$$0 \stackrel{!}{=} \left[\left. \frac{\delta \mathcal{S}(\Gamma)}{\delta\varphi_{k_1} \dots \delta\varphi_{k_N}} \right|_{\varphi=0} \right]_{\text{Terms}\propto p^{4-d}}, \tag{3.16}$$

where p denotes generically all appearing momenta. From power counting it is easy to see that every term in this identity can behave at most as p^{4-d} (up to logarithms). Hence the relevant terms are the leading terms in the momenta. In order to extract the necessary information, the identity can be evaluated in the limit $p \to \infty$ and all mass parameters can be neglected. This is an enormous simplification.

On the other hand, only counterterms of dimension = 4 contribute and can be determined using identities of this kind. Thus, no symmetry-breaking effects do contribute to Eq. (3.16). Hence, f_0 and v can be neglected. Such identities and these simplifications have been discussed and used in Ref. [16] to derive the counterterms to all dimension = 4 gluon and gluino interactions.

dimΔ_j = 3: Δ_j need not have R-charge zero. It can originate from a term such as $(f+f_0)\Delta_j$ or $(H+v)\Delta_j$ which has dimension 4 and is R-invariant. The possibilities for the R-charge $n(\Delta_j)$ are: $0, \pm n_H, \pm n_f$. They will be discussed later on. Since the relevant identity is

$$0 \stackrel{!}{=} \left[\frac{\delta \mathcal{S}(\Gamma)}{\delta\varphi_{k_1}\ldots\delta\varphi_{k_N}}\bigg|_{\varphi=0} \right]_{\text{Terms}\propto p^{3-d}}, \tag{3.17}$$

the vertex functions have to be evaluated up to the subleading order in p and the masses cannot be neglected. Nevertheless, one simplification is possible: Since $\Delta^{(1)}$ is a polynomial in the mass parameters in DREG (see Item 2 in Section 3.1), it is sufficient to evaluate the leading terms in the Taylor expansion in dimensionful couplings and masses (e.g. using the mass-insertion method), neglecting $\log m$-terms. This simplification is particularly valuable in models like the MSSM with many masses and mixing parameters.

Let us now go through the cases for $n(\Delta_j)$ and discuss the effects of symmetry breaking. If $n(\Delta_j) = -n_f$ and if Δ_j contains ϵ_α, i.e. $\varphi_k = \epsilon_\alpha$, soft supersymmetry breaking appears in the Slavnov-Taylor identity. The relevant term in $\delta\mathcal{S}(\Gamma)/(\delta\varphi_{k_1}\ldots\delta\varphi_{k_N}) = 0$ reads

$$\frac{\delta s\chi^\alpha}{\delta\epsilon^\beta}\Gamma_{\varphi_{k+1}\cdots\varphi_l\chi_\alpha} = \sqrt{2}f_0\,\Gamma_{\varphi_{k+1}\cdots\varphi_l\chi_\beta}. \tag{3.18}$$

The fields $\varphi_{k+1}\ldots\varphi_l\chi_\beta$ have a total dimension of $1+d$ and R-charge zero. Thus $\Gamma_{\varphi_{k+1}\cdots\varphi_l\chi_\beta}$ can receive non-zero counterterm contributions and it contributes to Eq. (3.17) at the order p^{3-d}. Its prefactor $\delta s\chi^\alpha/\delta\epsilon^\beta = \sqrt{2}f_0\delta^\alpha_\beta$ is proportional to the soft-breaking parameter f_0. The situation is similar if $n(\Delta_j) = +n_f$ and $\varphi_k = \bar\epsilon^{\dot\alpha}$. In all other cases, the term (3.18) does not contribute in Eq. (3.17) at the considered order and to the determination of the counterterms.

If $n(\Delta_j) = \pm n_H$, vertex functions like $\Gamma_{\psi_L\psi_R}$ or $\Gamma_{\bar\psi_L\bar\psi_R}$ that are proportional to the Higgs VEV v can appear in Eq. (3.17). The counterterms to these vertex functions are then restricted by Eq. (3.17). The monomials Δ_j and $H\Delta_j$ (or $H^\dagger\Delta_j$) and the corresponding identities are related by R-invariance. It is sufficient to consider only one of them.

If $n(\Delta_j) = 0$, no symmetry-breaking parameters contribute to Eq. (3.17). In this case, the identity (3.17) restricts the counterterms to R-invariant and supersymmetric dimension 3 interactions, like the μ-term in the MSSM. In the model of Section 2, no such interactions are possible and it exists no Δ_j of dimension 3 that has $n(\Delta_j) = 0$.

dim$\Delta_j \le 2$: The discussion of this case is similar to the previous one. The different possibilities concerning the R-charge are obvious, so we do not present the details.

4 Evaluation and restoration of specific symmetry relations

In this section we present an explicit calculation of symmetry identities on the one-loop level and of their symmetry-restoring counterterms. To be specific, we consider

symmetry identities corresponding to a particularly direct and interesting consequence of symmetry breakings: the appearance of non-zero masses for electrons and selectrons and their interrelations.

In our model R-invariance is spontaneously broken and the electron mass is generated via the electron–Higgs Yukawa interaction. The tree-level value is given by $m_e = vy_{123}$. The selectron masses are related to the electron mass via softly broken supersymmetry as can be seen from the appearance of m_e and soft-breaking parameters in the selectron mass matrix (2.9). Furthermore, the electron mass appears in the supersymmetry transformation

$$s\psi_R^\alpha = \ldots - \sqrt{2}\epsilon^\alpha y_{123}\phi_L^\dagger(H^\dagger + v) = \ldots - \sqrt{2}\epsilon^\alpha m_e \phi_L^\dagger. \tag{4.1}$$

These relations are replaced by Ward and Slavnov-Taylor identities in higher orders which are of the form (3.11) and the strategy of Section 3 can be applied. In this section, all Green functions appearing in the Slavnov-Taylor identities are evaluated analytically, the one-loop breakings are calculated explicitly, and suitable symmetry-restoring counterterms are determined. For the explicit calculation, we use DRED and DREG with anticommuting γ_5 and adopt the 4-spinor notation introduced in Appendix C.

4.1 Electron mass relation

The relation between the electron mass and the electron–Higgs Yukawa coupling is expressed by the Ward identity

$$0 = P_L \frac{\delta^2 \mathcal{W}(\Gamma)}{\delta\Psi\delta\bar{\Psi}} P_L \bigg|_{\varphi=0} = P_L \left(\Gamma_{\Psi\bar{\Psi}} - \sqrt{2}v\Gamma_{\Psi\bar{\Psi}H_2} + 3f_0\Gamma_{\Psi\bar{\Psi}f_2} \right) P_L. \tag{4.2}$$

Eq. (4.2) describes the spontaneous breaking of R-invariance. H_2 and f_2 are defined by $H_2 = -\sqrt{2}\mathrm{i}\mathrm{Im}H$ and $f_2 = -\mathrm{i}\mathrm{Im}f$. This identity connects the electron-mass contribution of the self energy $P_L\Gamma_{\Psi\bar{\Psi}}P_L$ $(= -m_e P_L + \text{higher orders})$ to the Yukawa coupling $\Gamma_{\Psi\bar{\Psi}H_2}$. The vertex function $\Gamma_{\Psi\bar{\Psi}f_2}$ has no R-invariant and power-counting renormalizable contribution and vanishes at tree level. At higher orders it contains only finite loop diagrams. An explicit one-loop calculation shows that Eq. (4.2) is valid both in DRED and DREG (with anticommuting γ_5) and no symmetry-restoring counterterms are required. The latter result is in agreement with the validity of the full Ward identity $\mathcal{W}\Gamma = 0$ in DREG as discussed in Section 3.1.

4.2 Electron-selectron mass relation

Our main interest in this subsection is the relation between the electron and selectron masses since this relation is influenced by spontaneous and soft symmetry breaking and involves the relation (4.1) of the supersymmetry transformation. The relevant Slavnov-Taylor identity can be obtained from

$$\frac{\delta \mathcal{S}(\Gamma)}{\delta\phi_L^\dagger(-p)\delta\Psi(p)\delta\bar{\epsilon}} \bigg|_{\varphi=0} = 0 \tag{4.3}$$

and reads

$$
\begin{aligned}
0 = {} & \Gamma_{\Psi\bar\epsilon Y_{\phi_L}}\Gamma_{\phi_L^\dagger\phi_L} + \Gamma_{\Psi\bar\epsilon Y_{\phi_R^\dagger}}\Gamma_{\phi_L^\dagger\phi_R^\dagger} + \Gamma_{Y_{\bar{\tilde H}}\bar\epsilon}\Gamma_{\phi_L^\dagger\Psi\bar{\tilde H}} + \Gamma_{\phi_L^\dagger Y_{\bar\Psi}\bar\epsilon}\Gamma_{\Psi\bar\Psi} \\
& + \frac{\partial\int \mathrm{d}^4x\, s\bar\Xi}{\partial\bar\epsilon}\bigg|_{\varphi=0}\Gamma_{\phi_L^\dagger\Psi\bar\Xi}.
\end{aligned} \tag{4.4}
$$

Some of the Green functions in Eq. (4.4) can appear at tree level and can receive counterterm contributions, for others the tree-level and counterterm contributions vanish due to R-invariance and power counting. The meaning of (4.4) gets more transparent if we multiply it with the projectors $P_{L,R}$ and distinguish those two types of contributions. Multiplying Eq. (4.4) from left and right with P_L yields

$$
\begin{aligned}
0 = {} & P_L\left(\Gamma_{\Psi\bar\epsilon Y_{\phi_L}}\Gamma_{\phi_L^\dagger\phi_L} + \Gamma_{\phi_L^\dagger Y_{\bar\Psi}\bar\epsilon}\Gamma_{\Psi\bar\Psi} + \sqrt{2}f_0\Gamma_{\phi_L^\dagger\Psi\bar\Xi}\right)P_L \\
& + \text{loop contributions.}
\end{aligned} \tag{4.5}
$$

The "loop contributions" contain all Green functions that are given in terms of finite one-loop diagrams and do not involve counterterms due to R-invariance and power counting. This identity relates the electron and selectron self energies and the soft-breaking term $\propto f_0$. In lowest order it corresponds to the mass relation $(M_\phi^2)_{11} = m_e^2 + M_{11}^2$. The prefactors of the self energies are loop-corrected supersymmetry transformations (see also Refs. [15,16] for further discussion).

These prefactors are also restricted by the following Slavnov-Taylor identity which corresponds to the supersymmetry algebra,

$$
\frac{\delta\mathcal{S}(\Gamma)}{\delta\phi_L(-p)\delta\epsilon\delta\bar\epsilon\delta Y_{\phi_L}(p)}\bigg|_{\varphi=0} = 0. \tag{4.6}
$$

If we multiply Eq. (4.4) with P_L from the left and with P_R from the right, we obtain

$$
0 = P_L\left(\Gamma_{\phi_L^\dagger Y_{\bar\Psi}\bar\epsilon}\Gamma_{\Psi\bar\Psi}\right)P_R + \text{loop contributions.} \tag{4.7}
$$

Written in terms of Weyl spinors, this identity relates in particular $\Gamma_{\phi_L^\dagger Y_{\psi_R}\epsilon}$ to $\Gamma_{\bar\psi_L\bar\psi_R}$ and, thus, to the electron mass. This identity yields Eq. (4.1) in lowest order.

The identities obtained by multiplying Eq. (4.4) with the projector P_R from the left are less important and are discussed later.

Both identities (4.5) and (4.7) are similar to the ones in SQED. However, Eq. (4.5) contains an explicit soft supersymmetry breaking term $\Gamma_{\phi_L^\dagger\Psi\bar\Xi}$. Furthermore, the vertex functions in both identities receive contributions from the Higgs field. Because of these additional contributions, in particular the new Ξ-interactions, the breaking of Eqs. (4.5) and (4.7) is modified in DREG compared to the SQED case, as we will see later in Eq. (4.12).

In a next step, we want to gain more insight into the identity (4.4) by studying the possible contributions of the breaking term $\mathcal{S}(\Gamma_{\text{reg}})^{(1)} = \Delta^{(1)}$ according to the classification of Section 3. The relevant terms in $\Delta^{(1)}$ can be expanded in the following monomials:

$$
\begin{aligned}
\Delta^{(1)}|_{\phi_L^\dagger\bar\epsilon\Psi-\text{part}} = \int d^4x\, \phi_L^\dagger\bar\epsilon\Big(& a_1P_L + a_2P_L\Box + a_3P_R + a_4P_R\Box \\
& + a_5P_L\mathrm{i}\gamma^\mu\partial_\mu + a_6P_R\mathrm{i}\gamma^\mu\partial_\mu\Big)\Psi.
\end{aligned} \tag{4.8}
$$

	a_1	a_2	a_3	a_4	a_5	a_6
Dimension	2	4	2	4	3	3
R charge	0	0	$-n_f + n_H$	$-n_f + n_H$	n_H	n_f

Table 2: Dimension and R-charges of the different contributions to the symmetry breaking corresponding to the parameters $a_1, \ldots, a_6$ as defined in Eq. (4.8)

With this notation, evaluating the r.h.s. of Eq. (4.4) at the regularized level, yields

$$a_1 P_L - a_2 P_L p^2 + a_3 P_R - a_4 P_R p^2 + a_5 P_L \not{p} + a_6 P_R \not{p}. \tag{4.9}$$

The dimensions and R-charges of the monomials corresponding to $a_1, \ldots, a_6$ are given in Table 4.2. Following the results of Section 3.2, a_4 has to vanish since the R-charge has to vanish for dimension 4 monomials. Moreover, a_2 can be determined in the limit $p \to \infty$. In this limit, Eq. (4.4) becomes a simple relation between the ϕ_L and Ψ self-energies:

$$\begin{aligned} -a_2 P_L p^2 = & \; P_L \left(\Gamma_{\Psi\bar{\epsilon}Y_{\phi_L}} \Gamma_{\phi_L^\dagger \phi_L} + \Gamma_{\phi_L^\dagger Y_{\bar{\Psi}} \bar{\epsilon}} \Gamma_{\Psi\bar{\Psi}} \right)^{(1)}_{\text{reg}} P_L \\ & + \text{subleading contributions.} \end{aligned} \tag{4.10}$$

The soft-breaking term and the vertex functions involving the Higgs field are negligible in this limit.

For finite momentum p, we can extract from the identity (4.4) information about the remaining monomials $\propto a_{1,3,5,6}$. Because of R-symmetry, the part of the identity corresponding to a_5 contains no soft-breaking term $\propto f_0$. All other identities are affected by terms $\propto f_0$.

In the following, we discuss the evaluation of the identity (4.4) in DRED and DREG at one-loop order. The explicit results for the vertex functions can be found in Appendix D.

In DRED, the regularized loop integrals can be reduced to a linear combination of standard scalar one- and two-point integrals, A_0 and B_0 functions, multiplied by rational functions of masses and external momenta (see Ref. [24] for conventions). The scalar A_0 and B_0 integrals contain logarithms of masses and momenta. The logarithms have to cancel within the identity since they cannot contribute to local (polynomial) breaking terms as required by the Quantum Action Principle. These non-local parts are functions of the arguments of the scalar integrals. Assuming that A_0 and B_0 integrals depending on different, non-vanishing arguments are uniquely characterized by their non-local parts, we expect that the coefficients in front of the A_0 and B_0 integrals depending on a certain set of non-vanishing arguments add up to zero in a symmetry identity. This is what happens in the identities considered above when DRED is used. As a result no symmetry-restoring counterterms are required in this identity using DRED.

This idea is also applicable for more complicated symmetry identities. After reducing the tensor integrals to scalar integrals, we can choose a basis of scalar integrals in the sense that these one-loop integrals are uniquely characterized by their non-local parts, i.e. the logarithms and dilogarithms. We call two scalar integrals $\mathcal{T}_1$ and $\mathcal{T}_2$ linearly independent if they cannot be related by

$$\mathcal{P}_1 \mathcal{T}_1 + \mathcal{P}_2 \mathcal{T}_2 = \mathcal{P}_3, \tag{4.11}$$

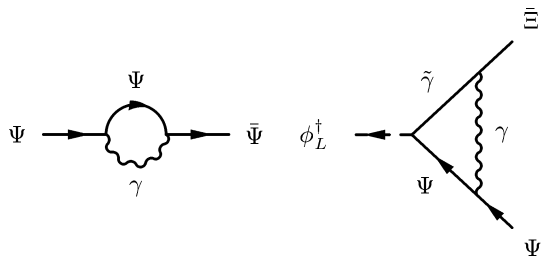

Figure 1: Diagrams contributing to the difference in the symmetry breaking (4.12) between DREG and DRED [see Eqs. (4.13) and (4.14)]

where $\mathcal{P}_{1,2,3}$ are polynomials of masses and momenta. These one-loop integrals then have to cancel within symmetry identities.

The calculation of one-loop diagrams may however yield rational functions of momenta and masses in addition to scalar integrals. These extra terms can originate from several sources. One is the reduction of tensor integrals to scalar ones, which is performed in D dimensions for DREG as well as DRED. Certain tensor integrals, which have at least two Lorentz indices, cannot be reduced to a combination of scalar integrals alone, e.g. in the tensor reduction of $B_{11}(p^2, m_0, m_1)$ a term $(m_0^2 + m_1^2 - p^2/3)/6$ remains apart from contributions involving scalar integrals. These integrals, however, do not occur in the calculation of the vertex functions of the r.h.s. of Eq. (4.4). Another source for polynomial terms in the calculation of symmetry identities, which contribute in DREG but not in DRED, is the appearance of terms $\propto (D-4)$ in the Dirac algebra, which multiply $1/(D-4)$-parts of the scalar integrals.

In the case of this particular identity, in DREG (with anticommuting γ_5) we obtain a finite rational function in the limit $D \to 4$ on the r.h.s. of Eq. (4.4):

$$\frac{\alpha}{4\pi}\Big[-\sqrt{2}(2m_e^2 + 2m_{\tilde\gamma}^2 - p^2)P_L - 2\sqrt{2}m_e m_{\tilde\gamma}P_R - \sqrt{2}\,(m_e P_L - m_{\tilde\gamma}P_R)\,\not{p}\Big]. \qquad (4.12)$$

We can directly read off the values of the coefficients $a_1, \ldots, a_6$, in particular $a_4 = 0$. The breaking term is in agreement with the Quantum Action Principle: It is a local term of dimension 2 and, as explained in Section 3.2, it is a polynomial in the mass parameters.

The appearance of the terms $\propto m_{\tilde\gamma} = -\tilde{M}_\lambda f_0$ shows that soft breaking of supersymmetry induces additional violations of the Slavnov-Taylor identity in DREG. In other words, DREG does not only violate supersymmetry, it even violates softly broken supersymmetry. Hence, in order to calculate supersymmetry-restoring counterterms, the soft-breaking must not be neglected.

The difference between DREG and DRED is confined to the following two vertex functions:

$$\Gamma^{(1)}_{\Psi\bar\Psi}(p,-p)|_{\mathrm{DREG-DRED}} = -\frac{\alpha}{4\pi}(\not{p} - 2m_e), \qquad (4.13)$$

$$\Gamma^{(1)}_{\phi_L^\dagger\Psi\bar\Xi}(p,-p,0)|_{\mathrm{DREG-DRED}} = \frac{\alpha}{4\pi}\frac{m_{\tilde\gamma}}{f_0}\left[\not{p} + 2(m_e - m_{\tilde\gamma})\right]P_L. \qquad (4.14)$$

The additional local contributions are generated in the diagrams shown in Figure 4.2. In particular the second diagram in Figure 4.2 involves a $\gamma\tilde\gamma\Xi$ vertex, i.e. an interaction of

the photon and two neutral fields. This does not correspond to a gauge interaction but to $\eta W^\alpha W_\alpha$ in Eq. (2.4), a type of interaction not present in a usual supersymmetric gauge theory without dimensionless chiral superfields. For this reason it is noteworthy that this new interaction induces additional symmetry violations in DREG, but not in DRED.

Identities like Eq. (4.6) have been discussed in Refs. [15,16] for SQED and SQCD. In order to extract the relevant restrictions on the supersymmetry transformations, we only need to evaluate these identities for $p \to \infty$ where soft breaking do not contribute. Hence these identities are respected by DRED and DREG.

4.3 Parameterization of counterterms

In this subsection, we parameterize the counterterms that appear in the Slavnov-Taylor identity (4.4). As mentioned in Section 3.1, the separation of the counterterms into symmetric and symmetry-restoring ones is not unique. It becomes unique if we impose that the symmetry-restoring counterterms do not contribute to those Green functions on which renormalization conditions are imposed. We make use of this possibility and denote the counterterms as follows (the momentum p is incoming into Ψ and outgoing from $\phi_L^\dagger$):

$$\Gamma^{\rm ct}_{\phi_L^\dagger \phi_L} = [p^2 - (M_\phi^2)_{11}](1+\delta Z_\phi) - \delta(M_\phi^2)_{11}, \tag{4.15}$$

$$\Gamma^{\rm ct}_{\phi_L^\dagger \phi_R^\dagger} = -(M_\phi^2)_{12} - \delta(M_\phi^2)_{12}, \tag{4.16}$$

$$\Gamma^{\rm ct}_{\Psi\bar\Psi} = (\not p - m_e)(1+\delta Z_\Psi) - \delta m_e, \tag{4.17}$$

$$\Gamma^{\rm ct}_{\phi_L^\dagger \Psi \bar{\tilde H}} = -P_R(y_{123} + \delta y_{123}), \tag{4.18}$$

$$\Gamma^{\rm ct}_{\Psi\bar\epsilon Y_{\phi_L}} = \sqrt{2} P_L (1 + \delta_{\psi\epsilon Y_\phi}), \tag{4.19}$$

$$\Gamma^{\rm ct}_{\Psi\bar\epsilon Y_{\phi_R^\dagger}} = -\sqrt{2} P_R (1 + \delta_{\psi\epsilon Y_\phi}), \tag{4.20}$$

$$\Gamma^{\rm ct}_{\phi_L^\dagger Y_{\bar\Psi}\bar\epsilon} = -\sqrt{2} P_L (\not p + m_e)(1 + \delta^1_{\phi\epsilon Y_\psi}) - \sqrt{2} P_L\, m_e\, \delta^2_{\psi\epsilon Y_\psi} - \sqrt{2} P_R\, f_0\, \delta u_{3\phi}, \tag{4.21}$$

$$\Gamma^{\rm ct}_{Y_{\bar{\tilde H}}\bar\epsilon} = \sqrt{2}\gamma_5 \frac{y_{333} v^2}{2} - \frac{\sqrt{2}}{2}\gamma_5 \left(2 f_0\, \delta u_{3H} - v^2 \delta y_{333}\right), \tag{4.22}$$

$$\Gamma^{\rm ct}_{\phi_L^\dagger \Psi \bar\Xi} = P_L \tilde M^2_{11}\, f_0 (1 + \delta^1_{\phi\psi\chi}) + P_R \tilde A_{123}\, v (1 + \delta^2_{\phi\psi\chi}) + \not p P_L\, \delta^3_{\phi\psi\chi}. \tag{4.23}$$

We have introduced the most general counterterms compatible with Lorentz invariance, R-symmetry, and power counting. Moreover, we have written down also the tree-level contributions. The renormalization constants δm_e, δM_ϕ^2, δZ_Ψ, δZ_ϕ, δy_{123}, $\delta u_{3\phi}$, δu_{3H} correspond to purely symmetric counterterms, whereas the constants $\delta_{\psi\epsilon Y_\phi}$, $\delta^{1,2}_{\phi\epsilon Y_\psi}$, $\delta^{1,2,3}_{\phi\psi\chi}$ correspond to a sum of symmetric and symmetry-restoring counterterms.

The parameters $\delta u_{3\phi}$, δu_{3H} deserve some discussion. They correspond to the u_3-parameters of Ref. [12]. These symmetric counterterms are generated by

$$\delta u_{3\phi}\, s_{\Gamma_{\rm cl}} \int {\rm d}^4x\, \bar\Xi\, P_R\, Y_{\bar\Psi}\, \phi_L^\dagger, \quad \delta u_{3H}\, s_{\Gamma_{\rm cl}} \int {\rm d}^4x\, \bar\Xi\, Y_{\bar{\tilde H}}. \tag{4.24}$$

In general, u_3-counterterms can be written as $\delta u_{3i}\, s_{\Gamma_{\rm cl}} \int {\rm d}^4x\, (Y_{\psi_i}\chi\phi_i)$, and adding these counterterms corresponds to a field renormalization of the form

$$\psi_{i_\alpha} \longrightarrow \psi_{i_\alpha} - \delta u_{3i} \chi_\alpha \phi_i. \tag{4.25}$$

The counterterms δu_{3i} do not contribute to physical amplitudes in the limit $\Xi = a = 0$ [12]. However, they are necessary in order to absorb divergences of vertex functions involving external fields. An example in this model is the contribution $P_R \Gamma_{\phi_L^\dagger Y_{\bar\Psi} \bar\epsilon} P_R$ since it does not appear in Γ_{cl}, but it contains divergences at one-loop order that have to be absorbed by $\delta u_{3\phi}$.

The δu_{3i}-parameters remain as undetermined constants in the symmetry identities and can be fixed by imposing renormalization conditions on the vertex functions $P_R \Gamma_{\phi_L^\dagger Y_{\bar\Psi} \bar\epsilon} P_R$ and $\Gamma_{Y_{\bar{\tilde{H}}} \bar\epsilon}$.

4.4 Results for the symmetry-restoring counterterms

In order to derive the consequences of the Slavnov-Taylor identity (4.4) for the counterterms, we rewrite Eq. (4.4) in the form of Eq. (3.14). Using the parameterization of the previous section, the breakings of Eq. (4.8) yield

$$\begin{aligned}
& a_1 P_L - a_2 P_L p^2 + a_3 P_R - a_4 P_R p^2 + a_5 P_L \not{p} + a_6 P_R \not{p} = \\
& \quad \sqrt{2} P_L\, \delta_{\psi\epsilon Y_\phi} \left[p^2 - (M_\phi^2)_{11}\right] + \sqrt{2} P_L \left\{ \left[(p^2 - (M_\phi^2)_{11}\right] \delta Z_\phi - \delta (M_\phi^2)_{11} \right\} \\
& \quad + \sqrt{2} P_R\, \delta_{\psi\epsilon Y_\phi} (M_\phi^2)_{12} + \sqrt{2} P_R\, \delta (M_\phi^2)_{12} \\
& \quad - \left[\sqrt{2} P_L (\not{p} + m_e) \delta^1_{\phi\epsilon Y_\psi} + \sqrt{2} P_L\, m_e \delta^2_{\psi\epsilon Y_\psi} + \sqrt{2} P_R\, f_0\, \delta u_{3\phi} \right] (\not{p} - m_e) \\
& \quad - \sqrt{2} P_L (\not{p} + m_e) \left[(\not{p} - m_e) \delta Z_\Psi - \delta m_e\right] \\
& \quad + \sqrt{2} \gamma_5\, f_0\, \delta u_{3H}\, P_R\, y_{123} - \sqrt{2} \gamma_5 \frac{y_{333} v^2}{2} P_R\, \delta y_{123} \\
& \quad - \sqrt{2} \gamma_5\, f_0 \left[P_L\, \tilde{M}_{11}^2\, f_0\, \delta^1_{\phi\psi\chi} + P_R\, \tilde{A}_{123}\, v\, \delta^2_{\phi\psi\chi} + \not{p} P_L\, \delta^3_{\phi\psi\chi} \right].
\end{aligned} \tag{4.26}$$

The l.h.s. vanishes for DRED and is given by Eq. (4.12) for DREG. The identity can be solved for $a_1, \ldots, a_6$ yielding

$$\begin{aligned}
\frac{a_1}{\sqrt{2}} &= -(M_\phi^2)_{11} (\delta_{\psi\epsilon Y_\phi} + \delta Z_\phi) - \delta (M_\phi^2)_{11} + m_e^2 (\delta^1_{\phi\epsilon Y_\psi} + \delta^2_{\psi\epsilon Y_\psi} + \delta Z_\Psi) \\
& \quad + m_e\, \delta m_e + f_0^2 \tilde{M}_{11}^2 \delta^1_{\phi\psi\chi},
\end{aligned} \tag{4.27}$$

$$\frac{a_2}{\sqrt{2}} = -\delta_{\psi\epsilon Y_\phi} - \delta Z_\phi + \delta^1_{\phi\epsilon Y_\psi} + \delta Z_\Psi, \tag{4.28}$$

$$\begin{aligned}
\frac{a_3}{\sqrt{2}} &= \delta_{\psi\epsilon Y_\phi} (M_\phi^2)_{12} + \delta (M_\phi^2)_{12} + m_e\, f_0\, \delta u_{3\phi} + y_{123}\, f_0\, \delta u_{3H} - \frac{y_{333} v^2}{2} \delta y_{123} \\
& \quad - f_0\, v\, \tilde{A}_{123}\, \delta^2_{\phi\psi\chi},
\end{aligned} \tag{4.29}$$

$$\frac{a_5}{\sqrt{2}} = -m_e \delta^2_{\psi\epsilon Y_\psi} + \delta m_e, \tag{4.30}$$

$$\frac{a_6}{\sqrt{2}} = -f_0\, \delta u_{3\phi} + f_0\, \delta^3_{\phi\psi\chi}. \tag{4.31}$$

Furthermore, the identity (4.6), which corresponds to the supersymmetry algebra, is respected by DRED as well as DREG. Hence, it yields in addition the relation

$$0 = \delta^1_{\phi\epsilon Y_\psi} + \delta_{\psi\epsilon Y_\phi}. \tag{4.32}$$

We have already discussed that the identities obtained by taking $P_L(\ldots)P_{L,R}$ of (4.4) describe several mass relations. If we restrict ourselves to these identities (they correspond to the parameters a_1, a_2, a_5) and combine them with Eq. (4.32), we obtain

$$\delta_{\psi\epsilon Y_\phi} = -\delta^1_{\phi\epsilon Y_\psi} = \frac{\delta Z_\Psi - \delta Z_\phi}{2} - \frac{a_2}{2\sqrt{2}}, \tag{4.33}$$

$$\delta^2_{\psi\epsilon Y_\psi} = \frac{\delta m_e}{m_e} - \frac{a_5}{\sqrt{2}m_e}, \tag{4.34}$$

$$\delta^1_{\phi\psi\chi} = \frac{\delta Z_\Psi + \delta Z_\phi}{2} + \frac{\delta(M^2_\phi)_{11} - 2m_e\,\delta m_e}{f_0^2 \tilde{M}^2_{11}} + \frac{a_1}{\sqrt{2}f_0^2\tilde{M}^2_{11}}. \tag{4.35}$$

All renormalization constants appearing in these identities are fixed except δZ_ϕ, δZ_Ψ, δm_e, $\delta(M^2_\phi)_{11}$. The latter correspond to symmetric counterterms, namely to the renormalization of the fields ϕ_L, Ψ and the mass parameters m_e, $(M^2_\phi)_{11}$. These counterterms are fixed by renormalization conditions to the electron and selectron self energies.

The results of Eqs. (4.29) and (4.31) corresponding to a_3 and a_6 can be solved similarly in order to obtain information on the counterterms $\delta^2_{\phi\psi\chi}$ and $\delta^3_{\phi\psi\chi}$. The solutions, however, involve many renormalization constants corresponding to symmetric counterterms, in particular to the u_3-parameters.

Although unphysical u_3-parameters appear in the identity (4.29), it is a relevant relation between the off-diagonal selectron mass $(M^2_\phi)_{12}$, the $\phi_L^\dagger\Psi\bar{\tilde{H}}$ Yukawa coupling, and the Ξ-interaction $P_R\Gamma_{\phi_L^\dagger\Psi\bar{\Xi}}P_R$. The latter interaction is particularly interesting since it originates from the term $\eta\Phi_1\Phi_2\Phi_3$ in $\mathcal{L}_{\text{soft}}$ (see Appendix A). Such an interaction is not present in usual renormalizable models where no dimensionless chiral superfields are present. In spite of this new interaction, the breaking term a_3 in Eq. (4.29) vanishes in DRED, but as expected it is non-zero in DREG.

Finally, we give the symmetry-restoring counterterm contributions to the Green functions in Eq. (4.4):

$$\Gamma^{\text{ct,restore}}_{\phi_L^\dagger\phi_L} = 0, \tag{4.36}$$

$$\Gamma^{\text{ct,restore}}_{\phi_L^\dagger\phi_R^\dagger} = 0, \tag{4.37}$$

$$\Gamma^{\text{ct,restore}}_{\Psi\bar{\Psi}} = 0, \tag{4.38}$$

$$\Gamma^{\text{ct,restore}}_{\phi_L^\dagger\Psi\bar{\tilde{H}}} = 0, \tag{4.39}$$

$$\Gamma^{\text{ct,restore}}_{\Psi\bar{\epsilon}Y_{\phi_L}} = -P_L\frac{a_2}{2}, \tag{4.40}$$

$$\Gamma^{\text{ct,restore}}_{\Psi\bar{\epsilon}Y_{\phi_R^\dagger}} = P_R\frac{a_2}{2}, \tag{4.41}$$

$$\Gamma^{\text{ct,restore}}_{\phi_L^\dagger Y_{\bar{\Psi}}\epsilon} = -P_L(\not{p} + m_e)\frac{a_2}{2} + P_L\,a_5, \tag{4.42}$$

$$\Gamma^{\text{ct,restore}}_{Y_{\bar{\tilde{H}}}\epsilon} = 0, \tag{4.43}$$

$$\Gamma^{\text{ct,restore}}_{\phi_L^\dagger\Psi\bar{\Xi}} = \frac{1}{\sqrt{2}f_0}(P_L\,a_1 - P_Ra_3 + \not{p}P_L\,a_6). \tag{4.44}$$

5 Summary and Conclusion

In this article we have considered a model with characteristic features of the MSSM, i.e. soft supersymmetry breaking and spontaneous symmetry breaking.

We have evaluated two important symmetry identities corresponding to mass relations for arbitrary momenta of the external fields in DREG as well as DRED. The Ward identity relating the electron mass to the Yukawa coupling is preserved both in DRED and in DREG. The Slavnov-Taylor identity relating the electron and selectron self energies is preserved in DRED, but violated in DREG.

We have shown that all appearing symmetry-restoring counterterms are uniquely fixed when the Slavnov-Taylor identity relating electron and selectron self energies is combined with an identity corresponding to the supersymmetry algebra.

Taking a closer look how the Slavnov-Taylor identity is violated in DREG reveals that the coefficients of all possible breaking terms [see Eq. (4.9)] are all non-vanishing. Moreover, in all cases where soft breaking is involved, soft breaking does actually affect the symmetry-restoring counterterms. In particular, the spurion field η and its spinor component Ξ give rise to $\eta W^\alpha W_\alpha$ and $\gamma\tilde{\gamma}\Xi$ interactions, which contribute to the violations of the Slavnov-Taylor identity in DREG, but not in DRED.

These results have important implications for the MSSM, where e.g. the left-handed stop and sbottom masses $m_{\tilde{t}_L}$, $m_{\tilde{b}_L}$ are related to the Ξ-interactions $\tilde{t}_L^\dagger t\bar{\Xi}$ and $\tilde{b}_L^\dagger b\bar{\Xi}$. These Ξ-interactions can be eliminated using SU(2) invariance and an identity between $m^2_{\tilde{t}_L} - m^2_{\tilde{b}_L}$ and $m_t^2 - m_b^2$ can be derived. Assuming that the results of the model considered here hold also for the MSSM, the identity for $m^2_{\tilde{t}_L} - m^2_{\tilde{b}_L}$, which is an important prediction of the MSSM, is fulfilled in DRED, but violated in DREG. Indeed the full (electroweak and strong) one-loop results of this relation evaluated in DRED and DREG without symmetry-restoring counterterms differ by finite contributions which are of the order of the full electroweak corrections [23].

Furthermore, several simplifications in the determination of symmetry-restoring counterterms have been discussed. The breaking Δ of a symmetry identity is a polynomial not only in the momenta, but also in the mass parameters of the model. Symmetry identities that correspond to dimension = 4 breakings can be evaluated in the limit $p \to \infty$, $m \to 0$. In symmetry identities corresponding to dimension < 4 breakings, masses and soft supersymmetry breaking cannot be neglected. However, in order to compute symmetry violations and symmetry-restoring counterterms, it is sufficient to evaluate the identities in a Taylor expansion in the masses. This is an important simplification in the restoration of symmetry identities in the MSSM where the structure of masses and mixings are rather complicated.

Appendix

A Lagrangian

The Lagrangian $\mathcal{L}$ is obtained from $\mathcal{L}_{\rm susy} + \mathcal{L}_{\rm soft}$ by choosing the Wess-Zumino gauge and eliminating the auxiliary fields. We specify the Lagrangian of the model in terms of mass eigenstates. The mass eigenstates and their eigenvalues are

$$\begin{aligned} \phi_1 &= \tfrac{1}{\sqrt{2}}(\phi_L + \phi_R^\dagger),\ M_1^2 = (y^{123})^2 v^2 + \tilde{M}_{11}^2 f_0^2 + \tfrac{1}{2} y^{123} y^{333} v^2 + 6 f_0 \tilde{A}_{123} v, \\ \phi_2 &= \tfrac{1}{\sqrt{2}}(\phi_R - \phi_L^\dagger),\ M_2^2 = (y^{123})^2 v^2 + \tilde{M}_{11}^2 f_0^2 - \tfrac{1}{2} y^{123} y^{333} v^2 - 6 f_0 \tilde{A}_{123} v \end{aligned} \tag{A.1}$$

as well as

$$\begin{aligned} H_1 &= \tfrac{1}{\sqrt{2}}(H + H^\dagger) = H_1^\dagger, & \tfrac{1}{2} M_{H_1}^2 &= \tfrac{3}{4}(y^{333})^2 v^2 + \tfrac{1}{2}\tilde{M}_{33}^2 f_0^2 + 3 f_0 \tilde{A}_{333} v, \\ H_2 &= \tfrac{1}{\sqrt{2}}(-H + H^\dagger) = -H_2^\dagger, & \tfrac{1}{2} M_{H_2}^2 &= \tfrac{1}{4}(y^{333})^2 v^2 + \tfrac{1}{2}\tilde{M}_{33}^2 f_0^2 - 3 f_0 \tilde{A}_{333} v. \end{aligned} \tag{A.2}$$

The Lagrangian of the model in the limit $a = \Xi = f = 0$ reads

$$\begin{aligned} &\mathcal{L}|_{a=\Xi=f=0} = \\ &\quad - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \bar{\tilde{\gamma}} \mathrm{i} \gamma^\mu \partial_\mu \tilde{\gamma} + \frac{1}{2} f_0\ \tilde{M}_\lambda \bar{\tilde{\gamma}} \tilde{\gamma} \\ &\quad + \bar{\Psi} \mathrm{i} \gamma^\mu D_\mu \Psi - y^{123} v \bar{\Psi} \Psi \\ &\quad + \frac{1}{2} \bar{\hat{H}} \mathrm{i} \gamma^\mu \partial_\mu \hat{H} - \frac{1}{2} y^{333} v \bar{\hat{H}} \hat{H} \\ &\quad + |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 \\ &\quad - \left[(y^{123})^2 v^2 + \tilde{M}_{11}^2 f_0^2 + 6 f_0\ \tilde{A}_{123} v + \frac{1}{2} y^{123} y^{333} v^2 \right] |\phi_1|^2 \\ &\quad - \left[(y^{123})^2 v^2 + \tilde{M}_{11}^2 f_0^2 - 6 f_0\ \tilde{A}_{123} v - \frac{1}{2} y^{123} y^{333} v^2 \right] |\phi_2|^2 \\ &\quad + \frac{1}{2} (\partial^\mu H_1^\dagger)(\partial_\mu H_1) + \frac{1}{2} (\partial^\mu H_2^\dagger)(\partial_\mu H_2) \\ &\quad - \frac{1}{2} \left[\frac{3}{2} (y^{333})^2 v^2 + \tilde{M}_{33} f_0^2 + 6 v f_0 \tilde{A}_{333} \right] |H_1|^2 \\ &\quad - \frac{1}{2} \left[\frac{1}{2} (y^{333})^2 v^2 + \tilde{M}_{33} f_0^2 - 6 v f_0 \tilde{A}_{333} \right] |H_2|^2 \\ &\quad - e Q_L \left(\bar{\Psi} \gamma_5 \tilde{\gamma} \phi_1 - \bar{\Psi} \tilde{\gamma} \phi_2^\dagger - \phi_1^\dagger \bar{\tilde{\gamma}} \gamma_5 \Psi - \phi_2 \bar{\tilde{\gamma}} \Psi \right) \\ &\quad - \frac{1}{\sqrt{2}} y^{123} \left[H_1 (\bar{\Psi} \Psi) + H_2 (\bar{\Psi} \gamma_5 \Psi) - \phi_2 (\bar{\hat{H}} \gamma_5 \Psi) + \phi_1^\dagger (\bar{\hat{H}} \Psi) + \phi_1 (\bar{\Psi} \hat{H}) + \phi_2^\dagger (\bar{\Psi} \gamma_5 \hat{H}) \right] \\ &\quad - \frac{1}{2\sqrt{2}} y^{333} [H_1 (\bar{\hat{H}} \hat{H}) + H_2 (\bar{\hat{H}} \gamma_5 \hat{H})] \\ &\quad - \frac{1}{\sqrt{2}} \left[2 (y^{123})^2 v + y^{123} y^{333} v + 6 f_0 \tilde{A}_{123} \right] H_1 |\phi_1|^2 \\ &\quad - \frac{1}{\sqrt{2}} \left[2 (y^{123})^2 v - y^{123} y^{333} v - 6 f_0 \tilde{A}_{123} \right] H_1 |\phi_2|^2 \end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{\sqrt{2}}\left(y^{123}y^{333}v-6f_0\tilde{A}_{123}\right)H_2\phi_1\phi_2\\
&-\frac{1}{\sqrt{2}}\left(-y^{123}y^{333}v+6f_0\tilde{A}_{123}\right)H_2\phi_1^\dagger\phi_2^\dagger\\
&-\frac{1}{\sqrt{2}}\left[\frac{1}{2}(y^{333})^2v+f_0\tilde{A}_{333}\right]H_1|H_1|^2-\frac{1}{\sqrt{2}}\left[\frac{1}{2}(y^{333})^2v-3f_0\tilde{A}_{333}\right]H_1|H_2|^2\\
&-\left[\frac{1}{2}e^2-\frac{1}{4}(y^{123})^2\right]\phi_1\phi_1\phi_2\phi_2-\left[\frac{1}{2}e^2-\frac{1}{4}(y^{123})^2\right]\phi_1^\dagger\phi_1^\dagger\phi_2^\dagger\phi_2^\dagger\\
&-e^2|\phi_1|^2|\phi_2|^2-\frac{1}{4}(y^{123})^2\left(|\phi_1|^4+|\phi_2|^4\right)\\
&+\left[-\frac{1}{2}(y^{123})^2-\frac{1}{4}y^{123}y^{333}\right]\left(|\phi_1|^2|H_1|^2+|\phi_2|^2|H_2|^2\right)\\
&+\left[-\frac{1}{2}(y^{123})^2+\frac{1}{4}y^{123}y^{333}\right]\left(|\phi_1|^2|H_2|^2+|\phi_2|^2|H_1|^2\right)\\
&-\frac{1}{2}y^{123}y^{333}H_1H_2(\phi_1\phi_2-\phi_1^\dagger\phi_2^\dagger)\\
&-\frac{1}{16}(y^{333})^2\left(|H_1|^4+2|H_1|^2|H_2|^2+|H_2|^4\right)\\
&-\left[\frac{1}{\sqrt{2}}v^3(y^{333})^2+3\sqrt{2}v^2f_0\tilde{A}_{333}+\sqrt{2}\tilde{M}_{33}^2f_0^2v\right]H_1\\
&-\frac{1}{4}(y^{333})^2v^4-2f_0\tilde{A}_{333}v^3-\tilde{M}_{33}^2f_0^2v^2.
\end{aligned}
\tag{A.3}
$$

In Eq. (A.3) we have transformed the scalar fields to their mass eigenstates, used four-component spinors for brevity, and introduced the covariant derivative D_μ and the electromagnetic field strength tensor $F_{\mu\nu}$:

$$
\begin{aligned}
D_\mu &= \partial_\mu + \mathrm{i}eQA_\mu,\\
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu.
\end{aligned}
\tag{A.4}
$$

All parameters in the Lagrangian $\mathcal{L}|_{a=\Xi=f=0}$ are real. For the evaluation of Slavnov-Taylor identities, it is necessary to know the parts of the Lagrangian that are linear in the fields Ξ and f. These parts of the Lagrangian are given by

$$
\begin{aligned}
&\mathcal{L}|_{\Xi-\mathrm{part}} =\\
&\frac{1}{\sqrt{2}}\left(\tilde{M}_{11}^2f_0+6\tilde{A}_{123}v\right)\left[\phi_1\bar{\Psi}\Xi+\phi_1^\dagger\bar{\Xi}\Psi\right]\\
&+\frac{1}{\sqrt{2}}\left(\tilde{M}_{11}^2f_0-6\tilde{A}_{123}v\right)\left[\phi_2\bar{\Xi}(P_R-P_L)\Psi+\phi_2^\dagger\bar{\Psi}(P_L-P_R)\Xi\right]\\
&+\frac{1}{\sqrt{2}}\left(\tilde{M}_{33}^2f_0+6\tilde{A}_{333}v\right)\bar{\Xi}\hat{H}H_1\\
&+\frac{1}{\sqrt{2}}\left(\tilde{M}_{33}^2f_0-6\tilde{A}_{333}v\right)\bar{\Xi}(P_L-P_R)\hat{H}H_2\\
&+\frac{\mathrm{i}}{2\sqrt{2}}\tilde{M}_\lambda\bar{\Xi}(P_R-P_L)\gamma^\nu\gamma^\mu\tilde{\gamma}F_{\mu\nu}
\end{aligned}
$$

$$
\begin{aligned}
&+3\tilde{A}_{123}\Big[(\bar{\Xi}\Psi)\left(\phi_1^\dagger H_1-\phi_2 H_2\right)+(\bar{\Xi}(P_R-P_L)\Psi)\left(\phi_1^\dagger H_2-\phi_2 H_1\right)\\
&\qquad+(\bar{\Psi}\Xi)\left(\phi_1 H_1-\phi_2^\dagger H_2\right)+(\bar{\Psi}(P_R-P_L)\Xi)\left(\phi_1 H_2-\phi_2^\dagger H_1\right)\\
&\qquad+\bar{\Xi}\hat{H}\left(\phi_1^\dagger\phi_1-\phi_2^\dagger\phi_2\right)+\bar{\Xi}(P_L-P_R)\hat{H}\left(\phi_1^\dagger\phi_2^\dagger-\phi_1\phi_2\right)\Big]\\
&-\frac{1}{\sqrt{2}}e\tilde{M}_\lambda(\bar{\Xi}\tilde{\gamma})(\phi_1\phi_2+\phi_1^\dagger\phi_2^\dagger)\\
&+\mathcal{O}(a,a^\dagger),
\end{aligned}
\tag{A.5}
$$

and

$$
\begin{aligned}
&\mathcal{L}|_{f-\mathrm{part}}=\\
&-\left(2f_0\tilde{M}_{11}^2+6\tilde{A}_{123}v\right)|\phi_1|^2 f_1-\left(2f_0\tilde{M}_{11}^2-6\tilde{A}_{123}v\right)|\phi_2|^2 f_1\\
&-\frac{1}{2}\left(2f_0\tilde{M}_{33}^2+6\tilde{A}_{333}v\right)|H_1|^2 f_1-\frac{1}{2}\left(2f_0\tilde{M}_{33}^2-6\tilde{A}_{333}v\right)|H_2|^2 f_1\\
&+6\tilde{A}_{123}v\phi_1\phi_2 f_2-6\tilde{A}_{123}v\phi_1^\dagger\phi_2^\dagger f_2-6\tilde{A}_{333}vH_1H_2f_2\\
&+\frac{1}{2}\tilde{M}_\lambda f_1\bar{\tilde{\gamma}}\tilde{\gamma}-\frac{1}{2}\tilde{M}_\lambda f_2\bar{\tilde{\gamma}}\gamma_5\tilde{\gamma}\\
&+\mathcal{O}(a,a^\dagger),
\end{aligned}
\tag{A.6}
$$

where we used Eq. (2.5) and

$$
f_{1,2}=\frac{1}{2}\left(\pm f+f^\dagger\right). \tag{A.7}
$$

The gauge fixing and ghost terms are combined in a BRS variation

$$
\begin{aligned}
\mathcal{L}_{\mathrm{fix,gh}}&=s\left[\bar{c}\left(\partial^\mu A_\mu+\frac{\xi}{2}B\right)\right]\\
&=B\partial^\mu A_\mu+\frac{\xi}{2}B^2-\bar{c}\Box c-\bar{c}\partial^\mu(\mathrm{i}\epsilon\sigma_\mu\bar{\lambda}-\mathrm{i}\lambda\sigma_\mu\bar{\epsilon})+\xi\mathrm{i}\epsilon\sigma^\nu\bar{\epsilon}(\partial_\nu\bar{c})\bar{c}.
\end{aligned}
\tag{A.8}
$$

When the auxiliary field B is eliminated, Eq. (A.8) yields the usual gauge-fixing term $-(\partial_\mu A^\mu)^2/(2\xi)$ and ghost terms, in particular the $\bar{c}\epsilon^\alpha\lambda_\alpha$ vertex due to the supersymmetry breaking of the gauge fixing.

The BRS transformations given in Appendix B that are non linear in the dynamical fields are coupled to external sources,

$$
\begin{aligned}
\mathcal{L}_{\mathrm{ext}}&=Y_\lambda^\alpha s\lambda_\alpha+Y_{\bar{\lambda}\dot{\alpha}}s\bar{\lambda}^{\dot{\alpha}}\\
&\qquad+Y_{\phi_L}s\phi_L+Y_{\phi_L^\dagger}s\phi_L^\dagger+Y_{\psi_L}^\alpha s\psi_{L\alpha}+Y_{\bar{\psi}_L\dot{\alpha}}s\bar{\psi}_L^{\dot{\alpha}}+(L\to R)\\
&\qquad+Y_{\tilde{H}}^\alpha s\tilde{H}_\alpha+Y_{\bar{\tilde{H}}\dot{\alpha}}s\bar{\tilde{H}}^{\dot{\alpha}}\\
&\qquad+\frac{1}{2}(Y_\lambda^\alpha\epsilon_\alpha+Y_{\bar{\lambda}\dot{\alpha}}\bar{\epsilon}^{\dot{\alpha}})^2-2(Y_{\tilde{H}}^\alpha\epsilon_\alpha)(\bar{\epsilon}_{\dot{\alpha}}Y_{\bar{\tilde{H}}}^{\dot{\alpha}})\\
&\qquad-2(Y_{\psi_L}^\alpha\epsilon_\alpha)(\bar{\epsilon}_{\dot{\alpha}}Y_{\bar{\psi}_L}^{\dot{\alpha}})-2(Y_{\psi_R}^\alpha\epsilon_\alpha)(\bar{\epsilon}_{\dot{\alpha}}Y_{\bar{\psi}_R}^{\dot{\alpha}}).
\end{aligned}
\tag{A.9}
$$

The classical action is given by

$$
\Gamma_{\mathrm{cl}}=\int \mathrm{d}^4x\ (\mathcal{L}+\mathcal{L}_{\mathrm{fix,gh}}+\mathcal{L}_{\mathrm{ext}}) \tag{A.10}
$$

and satisfies the Slavnov-Taylor identity $S(\Gamma_{\mathrm{cl}})=0$.

B BRS transformations

The BRS transformations of the fields of the model have the following explicit form:

$$
\begin{aligned}
sA_\mu &= \partial_\mu c + \mathrm{i}\epsilon\sigma_\mu\bar{\lambda} - \mathrm{i}\lambda\sigma_\mu\bar{\epsilon} - \mathrm{i}\omega^\nu\partial_\nu A_\mu, && \text{(B.1)}\\
s\lambda^\alpha &= \frac{\mathrm{i}}{2}(\epsilon\sigma^{\rho\sigma})^\alpha F_{\rho\sigma} - \mathrm{i}\epsilon^\alpha e Q_L(|\phi_L|^2 - |\phi_R|^2) - \mathrm{i}\omega^\nu\partial_\nu\lambda^\alpha, && \text{(B.2)}\\
s\bar{\lambda}_{\dot\alpha} &= \frac{-\mathrm{i}}{2}(\bar{\epsilon}\bar{\sigma}^{\rho\sigma})_{\dot\alpha} F_{\rho\sigma} - \mathrm{i}\epsilon_{\dot\alpha} e Q_L(|\phi_L|^2 - |\phi_R|^2) - \mathrm{i}\omega^\nu\partial_\nu\bar{\lambda}_{\dot\alpha}, && \text{(B.3)}\\
s\phi_L &= -\mathrm{i}eQ_L c\phi_L + \sqrt{2}\epsilon\psi_L - \mathrm{i}\omega^\nu\partial_\nu\phi_L, && \text{(B.4)}\\
s\phi_L^\dagger &= +\mathrm{i}eQ_L c\phi_L^\dagger + \sqrt{2}\bar{\psi}_L\bar{\epsilon} - \mathrm{i}\omega^\nu\partial_\nu\phi_L^\dagger, && \text{(B.5)}\\
s\psi_L^\alpha &= -\mathrm{i}eQ_L c\psi_L^\alpha - \sqrt{2}\epsilon^\alpha y^{123}\phi_R^\dagger(H^\dagger + v) - \sqrt{2}\mathrm{i}(\bar{\epsilon}\bar{\sigma}^\mu)^\alpha D_\mu\phi_L \\
&\quad - \mathrm{i}\omega^\nu\partial_\nu\psi_L^\alpha, && \text{(B.6)}\\
s\bar{\psi}_{L\dot\alpha} &= +\mathrm{i}eQ_L c\bar{\psi}_{L\dot\alpha} + \sqrt{2}\bar{\epsilon}_{\dot\alpha} y^{123}\phi_R(H + v) + \sqrt{2}\mathrm{i}(\epsilon\sigma^\mu)_{\dot\alpha}(D_\mu\phi_L)^\dagger \\
&\quad - \mathrm{i}\omega^\nu\partial_\nu\bar{\psi}_{L\dot\alpha}, && \text{(B.7)}\\
s\phi_R &= -\mathrm{i}eQ_R c\phi_R + \sqrt{2}\epsilon\psi_R - \mathrm{i}\omega^\nu\partial_\nu\phi_R, && \text{(B.8)}\\
s\phi_R^\dagger &= +\mathrm{i}eQ_R c\phi_R^\dagger + \sqrt{2}\bar{\psi}_R\bar{\epsilon} - \mathrm{i}\omega^\nu\partial_\nu\phi_R^\dagger, && \text{(B.9)}\\
s\psi_R^\alpha &= -\mathrm{i}eQ_R c\psi_R^\alpha - \sqrt{2}\epsilon^\alpha y^{123}\phi_L^\dagger(H^\dagger + v) - \sqrt{2}\mathrm{i}(\bar{\epsilon}\bar{\sigma}^\mu)^\alpha D_\mu\phi_R \\
&\quad - \mathrm{i}\omega^\nu\partial_\nu\psi_R^\alpha, && \text{(B.10)}\\
s\bar{\psi}_{R\dot\alpha} &= +\mathrm{i}eQ_R c\bar{\psi}_{R\dot\alpha} + \sqrt{2}\bar{\epsilon}_{\dot\alpha} y^{123}\phi_L(H + v) + \sqrt{2}\mathrm{i}(\epsilon\sigma^\mu)_{\dot\alpha}(D_\mu\phi_R)^\dagger \\
&\quad - \mathrm{i}\omega^\nu\partial_\nu\bar{\psi}_{R\dot\alpha}, && \text{(B.11)}\\
s(H + v) &= \sqrt{2}\epsilon\tilde{H} - \mathrm{i}\omega^\nu\partial_\nu H, && \text{(B.12)}\\
s(H^\dagger + v) &= \sqrt{2}\bar{\tilde{H}}\bar{\epsilon} - \mathrm{i}\omega^\nu\partial_\nu H^\dagger, && \text{(B.13)}\\
s\tilde{H}^\alpha &= -\sqrt{2}\epsilon^\alpha\left[y^{123}\phi_L^\dagger\phi_R^\dagger + \frac{1}{2}y^{333}(H^\dagger + v)(H^\dagger + v)\right] \\
&\quad - \sqrt{2}\mathrm{i}(\bar{\epsilon}\bar{\sigma}^\mu)^\alpha\partial_\mu H - \mathrm{i}\omega^\nu\partial_\nu\tilde{H}^\alpha, && \text{(B.14)}\\
s\bar{\tilde{H}}_{\dot\alpha} &= +\sqrt{2}\bar{\epsilon}_{\dot\alpha}\left[y^{123}\phi_L\phi_R + \frac{1}{2}y^{333}(H + v)(H + v)\right] \\
&\quad + \sqrt{2}\mathrm{i}(\epsilon\sigma^\mu)_{\dot\alpha}\partial_\mu H^\dagger - \mathrm{i}\omega^\nu\partial_\nu\bar{\tilde{H}}_{\dot\alpha}, && \text{(B.15)}\\
sc &= 2\mathrm{i}\epsilon\sigma^\nu\bar{\epsilon}A_\nu - \mathrm{i}\omega^\nu\partial_\nu c, && \text{(B.16)}\\
s\epsilon^\alpha &= 0, && \text{(B.17)}\\
s\bar{\epsilon}^{\dot\alpha} &= 0, && \text{(B.18)}\\
s\omega^\nu &= 2\epsilon\sigma^\nu\bar{\epsilon}, && \text{(B.19)}\\
s\bar{c} &= B - \mathrm{i}\omega^\nu\partial_\nu\bar{c}, && \text{(B.20)}\\
sB &= 2\mathrm{i}\epsilon\sigma^\nu\bar{\epsilon}\partial_\nu\bar{c} - \mathrm{i}\omega^\nu\partial_\nu B. && \text{(B.21)}
\end{aligned}
$$

The ghosts c, ϵ_α, $\bar{\epsilon}^{\dot\alpha}$, and ω^ν correspond to gauge, supersymmetry transformations, and translations. The antighost $\bar{c}$ and the auxiliary field B are needed for gauge fixing.

The BRS transformations of the fields of the external spurion multiplet are given by

$$
sa = \sqrt{2}\epsilon\chi - \mathrm{i}\omega^\nu\partial_\nu a, \qquad \text{(B.22)}
$$

$$
\begin{aligned}
sa^\dagger &= \sqrt{2}\bar{\chi}\bar{\epsilon} - \mathrm{i}\omega^\nu\partial_\nu a^\dagger, && \text{(B.23)}\\
s\chi^\alpha &= \sqrt{2}\epsilon^\alpha \hat{f} - \sqrt{2}\mathrm{i}(\bar{\epsilon}\bar{\sigma}^\mu)^\alpha\partial_\mu a - \mathrm{i}\omega^\nu\partial_\nu\chi^\alpha, && \text{(B.24)}\\
s\bar{\chi}_{\dot{\alpha}} &= -\sqrt{2}\bar{\epsilon}_{\dot{\alpha}}\hat{f}^\dagger + \sqrt{2}\mathrm{i}(\epsilon\sigma^\mu)_{\dot{\alpha}}\partial_\mu a^\dagger - \mathrm{i}\omega^\nu\partial_\nu\bar{\chi}_{\dot{\alpha}}, && \text{(B.25)}\\
sf &= \sqrt{2}\mathrm{i}\bar{\epsilon}\bar{\sigma}^\mu\partial_\mu\chi - \mathrm{i}\omega^\nu\partial_\nu f, && \text{(B.26)}\\
sf^\dagger &= -\sqrt{2}\mathrm{i}\partial_\mu\bar{\chi}\bar{\sigma}^\mu\epsilon - \mathrm{i}\omega^\nu\partial_\nu f^\dagger. && \text{(B.27)}
\end{aligned}
$$

C 4-spinor notation

We use the following conventions for derivatives with respect to Weyl spinors:

$$
\frac{\delta}{\delta\psi_\alpha}\psi_\beta = -\delta^\alpha{}_\beta, \quad \frac{\delta}{\delta\psi^\alpha}\psi^\beta = \delta_\alpha{}^\beta, \quad \frac{\delta}{\delta\bar{\psi}^{\dot{\alpha}}}\bar{\psi}^{\dot{\beta}} = -\delta_{\dot{\alpha}}{}^{\dot{\beta}}, \quad \frac{\delta}{\delta\bar{\psi}_{\dot{\alpha}}}\bar{\psi}^{\dot{\beta}} = \delta^{\dot{\alpha}}{}_{\dot{\beta}}. \tag{C.1}
$$

The 4-spinors and derivatives with respect to them are defined in such a way that $(\delta/\delta\Psi)\,\Psi = 1$ and $(\delta/\delta\bar{\Psi})\,\bar{\Psi} = 1$.

- Electron:

$$
\Psi = \begin{pmatrix} \psi_{L\alpha} \\ \bar{\psi}_R{}^{\dot{\alpha}} \end{pmatrix}, \qquad \frac{\delta}{\delta\Psi} = \begin{pmatrix} -\frac{\delta}{\delta\psi_{L\alpha}} & -\frac{\delta}{\delta\bar{\psi}_R{}^{\dot{\alpha}}} \end{pmatrix}, \tag{C.2}
$$

$$
\bar{\Psi} = \begin{pmatrix} \psi_R{}^\alpha & \bar{\psi}_{L\dot{\alpha}} \end{pmatrix}, \qquad \frac{\delta}{\delta\bar{\Psi}} = \begin{pmatrix} \frac{\delta}{\delta\psi_R^\alpha} \\ \frac{\delta}{\delta\bar{\psi}_{L\dot{\alpha}}} \end{pmatrix}. \tag{C.3}
$$

Sources for the BRS transformations of the electrons:

$$
Y_\Psi = \begin{pmatrix} Y^\alpha_{\psi_L} & Y_{\bar{\psi}_R\dot{\alpha}} \end{pmatrix}, \qquad \frac{\delta}{\delta Y_\Psi} = \begin{pmatrix} \frac{\delta}{\delta Y^\alpha_{\psi_L}} \\ \frac{\delta}{\delta Y_{\bar{\psi}_R\dot{\alpha}}} \end{pmatrix}, \tag{C.4}
$$

$$
Y_{\bar{\Psi}} = \begin{pmatrix} -Y_{\psi_R\alpha} \\ -Y^{\dot{\alpha}}_{\bar{\psi}_L} \end{pmatrix}, \qquad \frac{\delta}{\delta Y_{\bar{\Psi}}} = \begin{pmatrix} \frac{\delta}{\delta Y_{\psi_R\alpha}} & \frac{\delta}{\delta Y^{\dot{\alpha}}_{\bar{\psi}_L}} \end{pmatrix}. \tag{C.5}
$$

- Higgsino:

$$
\hat{H} = \begin{pmatrix} \tilde{H}_\alpha \\ \bar{\tilde{H}}{}^{\dot{\alpha}} \end{pmatrix}, \qquad \frac{\delta}{\delta\hat{H}} = \begin{pmatrix} -\frac{\delta}{\delta\tilde{H}_\alpha} & -\frac{\delta}{\delta\bar{\tilde{H}}{}^{\dot{\alpha}}} \end{pmatrix}, \tag{C.6}
$$

$$
Y_{\hat{H}} = \begin{pmatrix} Y^\alpha_{\tilde{H}} & Y_{\bar{\tilde{H}}\dot{\alpha}} \end{pmatrix}, \qquad \frac{\delta}{\delta Y_{\hat{H}}} = \begin{pmatrix} \frac{\delta}{\delta Y^\alpha_{\tilde{H}}} \\ \frac{\delta}{\delta Y_{\bar{\tilde{H}}\dot{\alpha}}} \end{pmatrix}. \tag{C.7}
$$

- Photino:

$$\tilde{\gamma} = \begin{pmatrix} -\mathrm{i}\lambda_\alpha \\ \mathrm{i}\bar{\lambda}^{\dot{\alpha}} \end{pmatrix}, \qquad \frac{\delta}{\delta\tilde{\gamma}} = \begin{pmatrix} -\mathrm{i}\frac{\delta}{\delta\lambda_\alpha} & \mathrm{i}\frac{\delta}{\delta\bar{\lambda}^{\dot{\alpha}}} \end{pmatrix}, \tag{C.8}$$

$$Y_{\tilde{\gamma}} = \begin{pmatrix} \mathrm{i}Y_\lambda^\alpha & -\mathrm{i}Y_{\bar{\lambda}\dot{\alpha}} \end{pmatrix}, \qquad \frac{\delta}{\delta Y_{\tilde{\gamma}}} = \begin{pmatrix} -\mathrm{i}\frac{\delta}{\delta Y_\lambda^\alpha} \\ \mathrm{i}\frac{\delta}{\delta Y_{\bar{\lambda}\dot{\alpha}}} \end{pmatrix}. \tag{C.9}$$

- Spinor component of the spurion superfield:

$$\Xi = \begin{pmatrix} \chi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}, \qquad \frac{\delta}{\delta\Xi} = \begin{pmatrix} -\frac{\delta}{\delta\chi_\alpha} & -\frac{\delta}{\delta\bar{\chi}^{\dot{\alpha}}} \end{pmatrix}. \tag{C.10}$$

- Supersymmetry ghost:

$$\epsilon = \begin{pmatrix} \epsilon_\alpha \\ \bar{\epsilon}^{\dot{\alpha}} \end{pmatrix}, \qquad \frac{\partial}{\partial\epsilon} = \begin{pmatrix} -\frac{\partial}{\partial\epsilon_\alpha} & -\frac{\partial}{\partial\bar{\epsilon}^{\dot{\alpha}}} \end{pmatrix}. \tag{C.11}$$

In this notation the Slavnov-Taylor operator is given by

$$\begin{aligned}
\mathcal{S}(\mathcal{F}) &= \mathcal{S}_0(\mathcal{F}) + \mathcal{S}_{\mathrm{soft}}(\mathcal{F}) \\
&= \int \mathrm{d}^4x \Bigg[sA^\mu \frac{\delta\mathcal{F}}{\delta A^\mu} + sc\frac{\delta\mathcal{F}}{\delta c} + s\bar{c}\frac{\delta\mathcal{F}}{\delta\bar{c}} + sB\frac{\delta\mathcal{F}}{\delta B} + \left(\frac{\delta\mathcal{F}}{\delta Y_{\tilde{\gamma}}}\right)^T \left(\frac{\delta\mathcal{F}}{\delta\tilde{\gamma}}\right)^T \\
&\qquad + \left(\frac{\delta\mathcal{F}}{\delta Y_\Psi}\right)^T \left(\frac{\delta\mathcal{F}}{\delta\Psi}\right)^T + \frac{\delta\mathcal{F}}{\delta Y_{\bar{\Psi}}}\frac{\delta\mathcal{F}}{\delta\bar{\Psi}} \\
&\qquad + \frac{\delta\mathcal{F}}{\delta Y_{\phi_L}}\frac{\delta\mathcal{F}}{\delta\phi_L} + \frac{\delta\mathcal{F}}{\delta Y_{\phi_L^\dagger}}\frac{\delta\mathcal{F}}{\delta\phi_L^\dagger} + \frac{\delta\mathcal{F}}{\delta Y_{\phi_R}}\frac{\delta\mathcal{F}}{\delta\phi_R} + \frac{\delta\mathcal{F}}{\delta Y_{\phi_R^\dagger}}\frac{\delta\mathcal{F}}{\delta\phi_R^\dagger} \\
&\qquad + s(H+v)\frac{\delta\mathcal{F}}{\delta H} + s(H^\dagger+v)\frac{\delta\mathcal{F}}{\delta H^\dagger} + \left(\frac{\delta\mathcal{F}}{\delta Y_{\hat{H}}}\right)^T \left(\frac{\delta\mathcal{F}}{\delta\hat{H}}\right)^T \Bigg] \\
&\quad + s\epsilon\frac{\partial\mathcal{F}}{\partial\epsilon} + s\omega^\nu\frac{\partial\mathcal{F}}{\partial\omega^\nu} \\
&\quad + \int \mathrm{d}^4x \Bigg[sa\frac{\delta\mathcal{F}}{\delta a} + sa^\dagger\frac{\delta\mathcal{F}}{\delta a^\dagger} + (s\Xi)^T \left(\frac{\delta\mathcal{F}}{\delta\Xi}\right)^T \\
&\qquad + s(f+f_0)\frac{\delta\mathcal{F}}{\delta f} + s(f^\dagger+f_0)\frac{\delta\mathcal{F}}{\delta f^\dagger}\Bigg].
\end{aligned} \tag{C.12}$$

The BRS transformations of the 4-spinors read

$$\begin{aligned}
s\Psi &= \mathrm{i}ec\Psi - \sqrt{2}y^{123}P_L\epsilon\phi_R^\dagger(H^\dagger+v) + \sqrt{2}y^{123}P_R\epsilon\phi_L(H+v) \\
&\quad + \sqrt{2}\mathrm{i}P_L\gamma^\mu\epsilon D_\mu\phi_L - \sqrt{2}\mathrm{i}P_R\gamma^\mu\epsilon D_\mu\phi_R^\dagger - \mathrm{i}\omega^\nu\partial_\nu\Psi,
\end{aligned} \tag{C.13}$$

$$\begin{aligned}
(s\bar{\Psi})^T &= -\mathrm{i}ec\bar{\Psi}^T + \sqrt{2}y^{123}P_R\bar{\epsilon}^T\phi_R(H+v) - \sqrt{2}y^{123}P_L\bar{\epsilon}^T\phi_L^\dagger(H^\dagger+v) \\
&\quad + \sqrt{2}\mathrm{i}\gamma^{\mu T}P_L\bar{\epsilon}^T D_\mu\phi_L^\dagger - \sqrt{2}\mathrm{i}\gamma^{\mu T}P_R\bar{\epsilon}^T D_\mu\phi_R - \mathrm{i}\omega^\nu\partial_\nu\bar{\Psi}^T,
\end{aligned} \tag{C.14}$$

$$s\tilde{\gamma} = \frac{\mathrm{i}}{4}[\gamma^\sigma,\gamma^\rho]\epsilon F_{\rho\sigma} + (P_R - P_L)\epsilon e Q_L(|\phi_L|^2 - |\phi_R|^2)$$

$$
\begin{aligned}
&+(P_R-P_L)\omega^\nu\partial_\nu\tilde{\gamma}, &\text{(C.15)}\\
s\hat{H} &= -\sqrt{2}P_L\epsilon\left[y^{123}\phi_L^\dagger\phi_R^\dagger+\frac{1}{2}y^{333}(H^\dagger+v)(H^\dagger+v)\right]\\
&+\sqrt{2}P_R\epsilon\left[y^{123}\phi_L\phi_R+\frac{1}{2}y^{333}(H+v)(H+v)\right]\\
&+\sqrt{2}\mathrm{i}\gamma^\mu P_R\epsilon\partial_\mu H^\dagger-\sqrt{2}\mathrm{i}\gamma^\mu P_L\epsilon\partial_\mu H-\mathrm{i}\omega^\nu\partial_\nu\hat{H}, &\text{(C.16)}\\
(s\Xi)^T &= \sqrt{2}\hat{f}\epsilon^T P_L-\sqrt{2}\hat{f}^\dagger\epsilon^T P_R-\sqrt{2}\mathrm{i}\epsilon^T\gamma^{\mu T}P_L\partial_\mu a+\sqrt{2}\mathrm{i}\epsilon^T\gamma^{\mu T}P_R\partial_\mu a^\dagger\\
&-\mathrm{i}\omega^\nu\partial_\nu\Xi^T. &\text{(C.17)}
\end{aligned}
$$

D One-loop results

We list the one-loop diagrams of the vertex functions of identity (4.4) as well as the results given as one-loop integrals. We choose $\xi=1$ in the gauge fixing term and use the following abbreviations:

$$
\begin{aligned}
d_{11} &= \frac{1}{\sqrt{2}v}\left(2m_e^2+\frac{m_e m_H}{2}+\frac{M_1^2-M_2^2}{2}\right), &\text{(D.1)}\\
d_{12} &= \frac{1}{\sqrt{2}v}\left(2m_e^2-\frac{m_e m_H}{2}-\frac{M_1^2-M_2^2}{2}\right), &\text{(D.2)}\\
d_{22} &= \frac{1}{\sqrt{2}v}\left(-\frac{3}{2}m_e m_H+\frac{M_1^2-M_2^2}{2}\right)=-d_{21}, &\text{(D.3)}\\
x_1 &= \frac{1}{\sqrt{2}f_0}\left(M_1^2-m_e^2-\frac{m_e m_H}{2}\right), &\text{(D.4)}\\
x_2 &= \frac{1}{\sqrt{2}f_0}\left(M_2^2-m_e^2+\frac{m_e m_H}{2}\right), &\text{(D.5)}\\
x_{H_1} &= \frac{1}{\sqrt{2}f_0}\left(M_{H_1}^2-\frac{3}{2}m_H^2\right), &\text{(D.6)}\\
x_{H_2} &= \frac{1}{\sqrt{2}f_0}\left(M_{H_2}^2-\frac{1}{2}m_H^2\right), &\text{(D.7)}
\end{aligned}
$$

and

$$
\theta_{\mathrm{DREG}}=\begin{cases}1 & \text{for DREG,}\\ 0 & \text{for DRED.}\end{cases} \qquad \text{(D.8)}
$$

For reasons of brevity we do not express B_1, C_0, C_1 integrals in terms of A_0 and B_0 integrals and omit the momentum arguments in the one-loop functions (see Ref. [24] for definitions):

$$
\begin{aligned}
C_{\{0,1\}}(m_0,m_1,m_2) &:= C_{\{0,1\}}(p,0,m_0,m_1,m_2), &\text{(D.9)}\\
B_{\{0,1\}}(m_0,m_1) &:= B_{\{0,1\}}(p,m_0,m_1). &\text{(D.10)}
\end{aligned}
$$

Furthermore, we often combine the explicit results of two Green functions into one equation where the upper sign of $\pm,\mp$ (first index) corresponds to the first Green function and the lower one (second index) to the second.

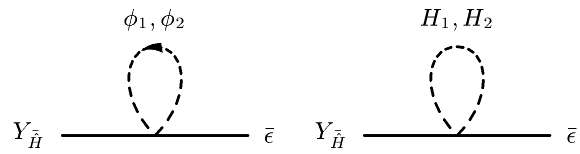

Figure 2: One-loop diagrams to $\Gamma_{Y_{\bar{\tilde{H}}}\bar{\epsilon}}$

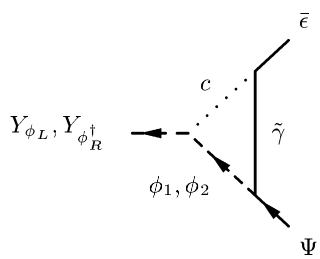

Figure 3: One-loop diagrams to $\Gamma_{\Psi\bar{\epsilon}Y_{\phi_L}}$ and $\Gamma_{\Psi\bar{\epsilon}Y_{\phi_R^\dagger}}$

$$
\begin{aligned}
&\Gamma^{(1)}_{Y_{\bar{\tilde{H}}}\bar{\epsilon}}(0,0) = \frac{\alpha}{4\pi}\frac{1}{\sqrt{2}e^2}\gamma_5\big\{ \\
&\quad y^{123}\left[A_0(M_2) - A_0(M_1)\right] + y^{333}\left[A_0(M_{H_2}) - A_0(M_{H_1})\right]\big\}, \qquad \text{(D.11)}\\
&\left\{\Gamma^{(1)}_{\Psi\bar{\epsilon}Y_{\phi_L}}(p,0,-p), \Gamma^{(1)}_{\Psi\bar{\epsilon}Y_{\phi_R^\dagger}}(p,0,-p)\right\} = \frac{\alpha}{4\pi}\frac{1}{\sqrt{2}}\big\{ \\
&\quad - \gamma_5\left[B_0(m_{\tilde{\gamma}}, M_1) + m_{\tilde{\gamma}}\, \not{p}\, C_1(m_{\tilde{\gamma}}, M_1, 0)\right] \\
&\quad \pm \left[B_0(m_{\tilde{\gamma}}, M_2) - m_{\tilde{\gamma}}\, \not{p}\, C_1(m_{\tilde{\gamma}}, M_2, 0)\right]\big\}, \qquad \text{(D.12)}\\
&\left\{\Gamma^{(1)}_{\phi_1^\dagger\phi_1}(-p,p), \Gamma^{(1)}_{\phi_2^\dagger\phi_2}(p,-p)\right\} = \frac{\alpha}{4\pi}\Bigg\{ \\
&\quad - \left[2(M^2_{1,2} + p^2)B_0(0, M_{1,2}) - A_0(M_{1,2})\right] \\
&\quad - \frac{1}{2}\left(\frac{y^{123}}{e}\right)^2\left[4p^2 B_1(m_{H,e}, m_{e,H}) + 4A_0(m_{e,H}) + 4(m^2_{H,e} \pm m_e m_H)B_0(m_H, m_e)\right] \\
&\quad - \left[4p^2 B_1(m_{\tilde{\gamma},e}, m_{e,\tilde{\gamma}}) + 4A_0(m_{e,\tilde{\gamma}}) + 4\left(\mp m_e m_{\tilde{\gamma}} + m^2_{\tilde{\gamma},e}\right)B_0(m_{\tilde{\gamma}}, m_e)\right] \\
&\quad + \frac{1}{e^2}\left[d^2_{11,12}B_0(M_{H_1}, M_{1,2}) + d^2_{21}B_0(M_{H_2}, M_{2,1})\right] \\
&\quad + \frac{1}{2e^2}\left[\left(\left(y^{123}\right)^2 \pm \frac{1}{2}y^{123}y^{333}\right)A_0(M_{H_1}) + \left(\left(y^{123}\right)^2 \mp \frac{1}{2}y^{123}y^{333}\right)A_0(M_{H_2})\right] \\
&\quad + A_0(M_{2,1}) + \left(\frac{y^{123}}{e}\right)^2 A_0(M_{1,2})\Bigg\}, \qquad \text{(D.13)}\\
&\left\{\Gamma^{(1)}_{\phi_1^\dagger\Psi\bar{\tilde{H}}}(-p,p,0), \Gamma^{(1)}_{\phi_2\Psi\bar{\tilde{H}}}(-p,p,0)\right\} = \frac{\alpha}{4\pi}\{1,\gamma_5\}\Bigg\{
\end{aligned}
$$

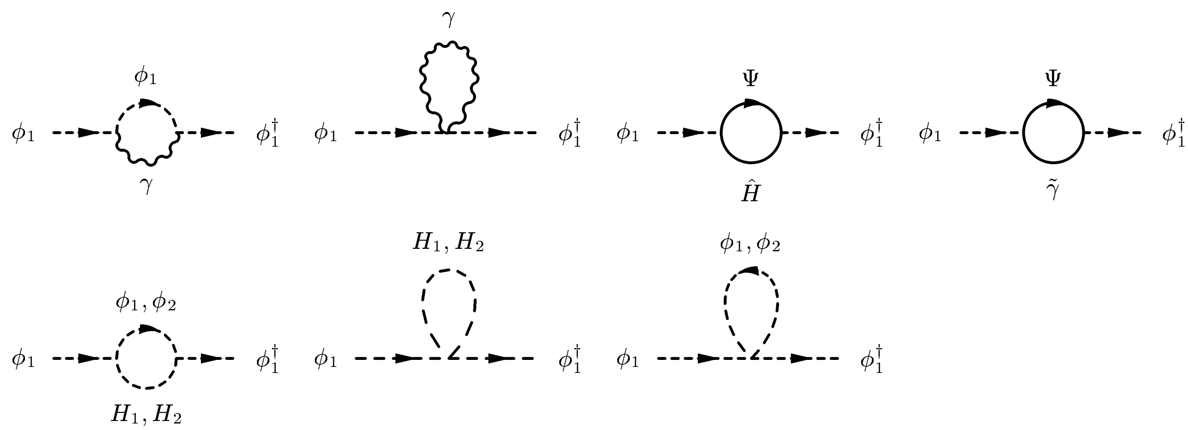

Figure 4: One-loop diagrams to $\Gamma_{\phi_1^\dagger \phi_1}$

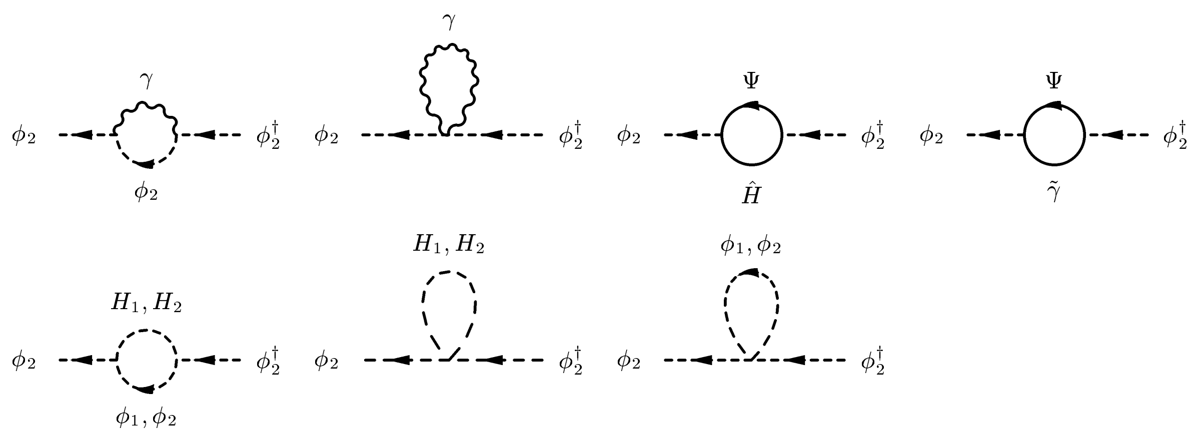

Figure 5: One-loop diagrams to $\Gamma_{\phi_2^\dagger \phi_2}$

$$
\begin{aligned}
&\pm \frac{1}{2}\left(\frac{y^{123}}{e}\right)^2 \big[- d_{11,12}\, \not{p}\, C_1(m_e, M_{H_1}, M_{1,2}) + d_{11,12}\, m_e\, C_0(m_e, M_{H_1}, M_{1,2}) \\
&\qquad \mp d_{21}\, \not{p}\, C_1(m_e, M_{H_2}, M_{2,1}) \mp d_{21}\, m_e\, C_0(m_e, M_{H_2}, M_{2,1}) \big], \\
&\pm \frac{1}{2}\frac{y^{123} y^{333}}{e^2} \big[\mp d_{11,12}\, \not{p}\, C_1(m_H, M_{1,2}, M_{H_1}) + d_{11,12}\, m_H\, C_0(m_H, M_{1,2}, M_{H_1}) \\
&\qquad - d_{21}\, \not{p}\, C_1(m_H, M_{2,1}, M_{H_2}) \mp d_{21}\, m_H\, C_0(m_H, M_{2,1}, M_{H_2}) \big] \\
&\pm \frac{\left(y^{123}\right)^2 y^{333}}{2\sqrt{2}\, e^2} \Big[\pm \big(p^2 + \not{p}\,(m_e \pm m_H)\big)\, C_1(m_H, m_e, M_{H_1}) \\
&\qquad \pm \frac{1}{M_{H_1}^2 - m_H^2} \big(M_{H_1}^2\, B_0(M_{H_1}, m_e) - m_H^2\, B_0(m_H, m_e) \big)
\end{aligned}
$$

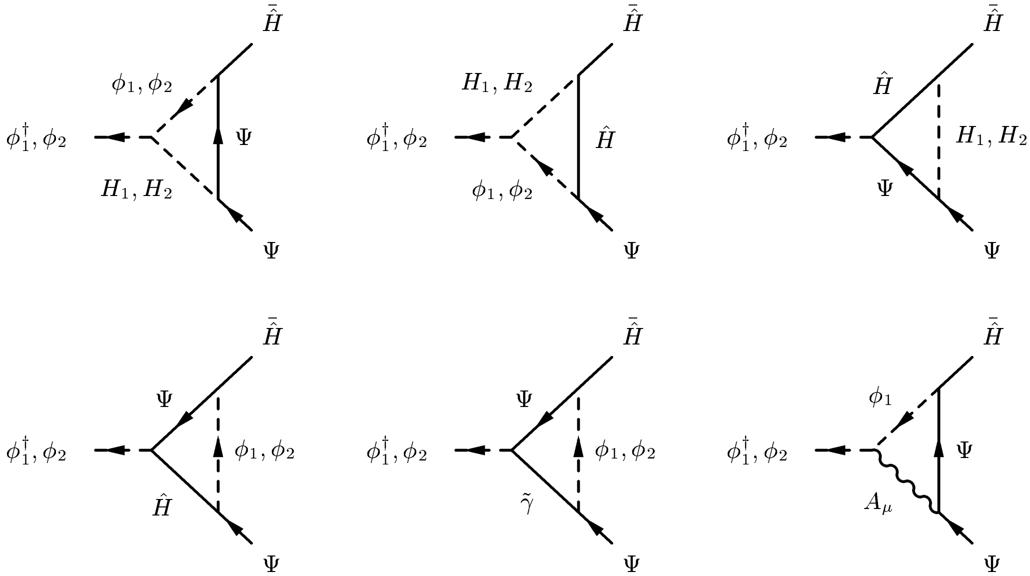

Figure 6: One-loop diagrams to $\Gamma_{\phi_1^\dagger \Psi \bar{\hat{H}}}$ and $\Gamma_{\phi_2 \Psi \bar{\hat{H}}}$

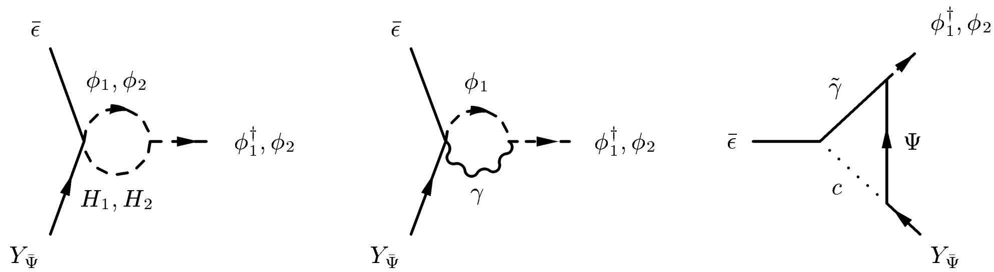

Figure 7: One-loop diagrams to $\Gamma_{\phi_1^\dagger Y_{\bar{\Psi}} \bar{\epsilon}}$ and $\Gamma_{\phi_2 Y_{\bar{\Psi}} \bar{\epsilon}}$

$$
\begin{aligned}
&+ \left(\not{p} m_H + m_e m_H\right) C_0(m_H, m_e, M_{H_1}) \\
&\mp \left(p^2 - \not{p}\,(m_e \pm m_H)\right) C_1(m_H, m_e, M_{H_2}) \\
&\mp \frac{1}{M_{H_2}^2 - m_H^2} \left(M_{H_2}^2\, B_0(M_{H_2}, m_e) - m_H^2\, B_0(m_H, m_e)\right) \\
&- \left(m_e m_H - \not{p} m_H\right) C_0(m_H, m_e, M_{H_2}) \Big]
\end{aligned}
$$

$$
\begin{aligned}
\pm \frac{(y^{123})^3}{2\sqrt{2}\, e^2} \Big[&\pm \left(p^2 + \not{p}\,(m_H \pm m_e)\right) C_1(m_e, m_H, M_1) \\
&\pm \frac{1}{M_1^2 - m_e^2} \left(M_1^2\, B_0(M_1, m_H) - m_e^2\, B_0(m_e, m_H)\right)
\end{aligned}
$$

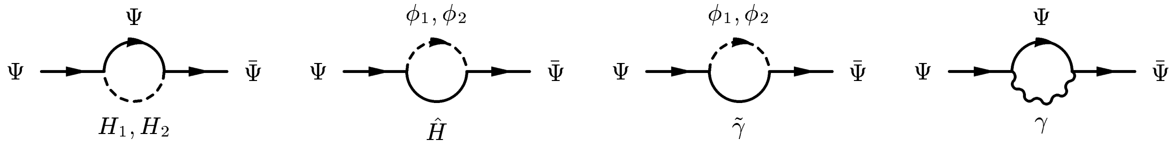

Figure 8: One-loop diagrams to $\Gamma^{(1)}_{\Psi\bar{\Psi}}$

$$
\begin{aligned}
&+ (\not{p} m_e + m_e m_H)\; C_0(m_e, m_H, M_1) \\
&\mp \left(p^2 - \not{p}\,(m_H \pm m_e)\right) C_1(m_e, m_H, M_2) \\
&\mp \frac{1}{M_2^2 - m_e^2} \left(M_2^2\, B_0(M_2, m_H) - m_e^2\, B_0(m_e, m_H)\right) \\
&- (m_e m_H - \not{p} m_e)\; C_0(m_e, m_H, M_2)\Big] \\
+ \frac{y^{123}}{\sqrt{2}} \Big[&\mp \left(p^2 - \not{p}\,(m_{\tilde\gamma} \mp m_e)\right) C_1(m_e, m_{\tilde\gamma}, M_1) \\
&\mp \frac{1}{M_1^2 - m_e^2} \left(M_1^2\, B_0(M_1, m_{\tilde\gamma}) - m_e^2\, B_0(m_e, m_{\tilde\gamma})\right) \\
&+ (-\not{p} m_e + m_e m_{\tilde\gamma})\; C_0(m_e, m_{\tilde\gamma}, M_1) \\
&\mp \left(p^2 + \not{p}\,(m_{\tilde\gamma} \mp m_e)\right) C_1(m_e, m_{\tilde\gamma}, M_2) \\
&\mp \frac{1}{M_2^2 - m_e^2} \left(M_2^2\, B_0(M_2, m_{\tilde\gamma}) - m_e^2\, B_0(m_e, m_{\tilde\gamma})\right) \\
&+ (m_e m_{\tilde\gamma} + \not{p} m_e)\; C_0(m_e, m_{\tilde\gamma}, M_2)\Big] \\
\mp\; \frac{y^{123}}{\sqrt{2}} \Big[&\frac{1}{2} \left(B_0(M_{1,2}, 0) + B_0(m_e, 0) + \left(m_e^2 + M_{1,2}^2\right)\; C_0(m_e, 0, M_{1,2})\right) \\
&+ \not{p} m_e \left(C_0(m_e, 0, M_{1,2}) - C_1(m_e, 0, M_{1,2})\right) \\
&-\; p^2 C_1(m_e, 0, M_{1,2})\Big]\Big\}, \qquad \text{(D.14)}
\end{aligned}
$$

$$
\begin{aligned}
&\left\{\Gamma^{(1)}_{\phi_1^\dagger Y_{\bar\Psi}\bar\epsilon}(-p,p,0), \Gamma^{(1)}_{\phi_2 Y_{\bar\Psi}\bar\epsilon}(-p,p,0)\right\} = \frac{\alpha}{4\pi}\{\gamma_5, 1\}\Big\{ \\
&\pm \frac{y^{123}}{\sqrt{2}\, e^2} \left[d_{21}\, B_0(M_{H_2}, M_{2,1}) \mp d_{11,12}\, B_0(M_{H_1}, M_{1,2})\right] \\
&- \left[2\not{p} B_0(0, M_{1,2}) + \not{p} B_1(0, M_{1,2})\right] \\
&\mp \big[\mp \not{p}\, B_1(m_{\tilde\gamma}, m_e) \mp (\not{p} + m_e \mp m_{\tilde\gamma})\; B_0(m_{\tilde\gamma}, m_e)
\end{aligned}
$$

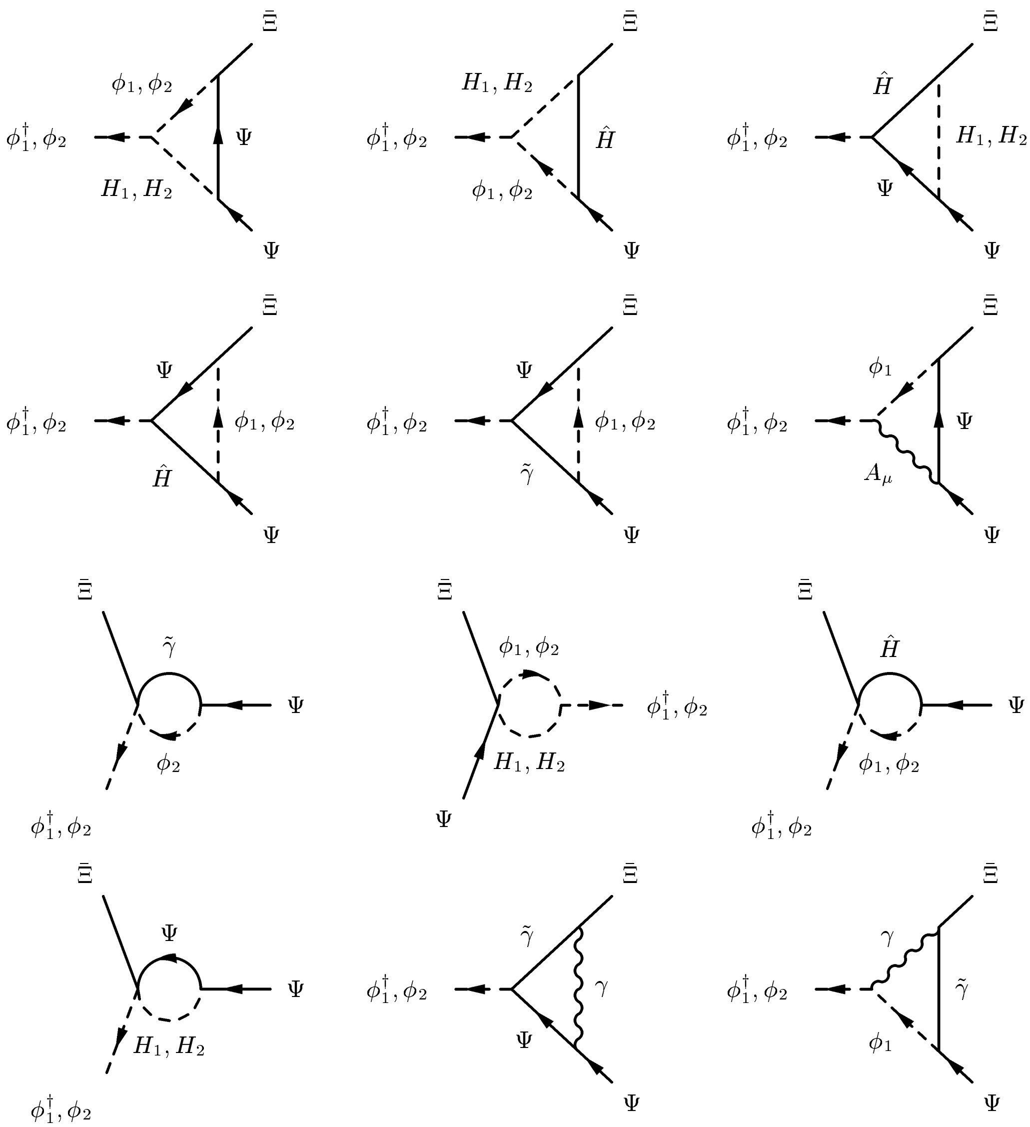

Figure 9: One-loop diagrams to $\Gamma_{\phi_1^\dagger \Psi \bar{\Xi}}$ and $\Gamma_{\phi_2 \Psi \bar{\Xi}}$

$$+ \left(m_{\tilde{\gamma}} p^2 + m_{\tilde{\gamma}} m_e \not{p}\right) \, C_1(m_{\tilde{\gamma}}, m_e, 0)\Big]\Big\}, \tag{D.15}$$

$$\Gamma^{(1)}_{\Psi\bar{\Psi}}(p,-p) = \frac{\alpha}{4\pi}\Big\{$$

$$\frac{1}{2}\left(\frac{y^{123}}{e}\right)^2 \Big[\not{p}\,(B_0(M_{H_1}, m_e) + B_1(M_{H_1}, m_e)) + m_e B_0(M_{H_1}, m_e)$$

$$+ \not{p}\,(B_0(M_{H_2}, m_e) + B_1(M_{H_2}, m_e)) - m_e B_0(M_{H_2}, m_e)\Big]$$

$$+ \frac{1}{2}\left(\frac{y^{123}}{e}\right)^2 \Big[\not{p}\,(B_0(M_1, m_H) + B_1(M_1, m_H)) + m_H B_0(M_1, m_H)$$

$$
\begin{aligned}
&\qquad + \not{p}\left(B_0(M_2,m_H)+B_1(M_2,m_H)\right) - m_H B_0(M_2,m_H)\big] \\
&+ \big[\not{p}\left(B_0(M_1,m_{\tilde\gamma})+B_1(M_1,m_{\tilde\gamma})\right) - m_{\tilde\gamma} B_0(M_1,m_{\tilde\gamma}) \\
&\quad + \not{p}\left(B_0(M_2,m_{\tilde\gamma})+B_1(M_2,m_{\tilde\gamma})\right) + m_{\tilde\gamma} B_0(M_2,m_{\tilde\gamma})\big] \\
&- \left[\left(4m_e - \not{p} - \not{p}\frac{m_e^2}{p^2}\right) B_0(0,m_e) + \frac{\not{p}}{p^2} A_0(m_e) + \theta_{\mathrm{DREG}}\left(\not{p} - 2m_e\right)\right]\Bigg\}, \qquad \text{(D.16)}
\end{aligned}
$$

$$
\begin{aligned}
&\Big\{\Gamma^{(1)}_{\phi_1^\dagger\Psi\bar\Xi}(-p,p,0), \Gamma^{(1)}_{\phi_2\Psi\bar\Xi}(-p,p,0)\Big\} = \frac{\alpha}{4\pi}\{1,\gamma_5\}\Bigg\{ \\
&- \frac{y^{123}}{\sqrt{2}\,e^2}\Big[\pm x_1\, d_{11,21}\ \left(\mp\not{p}\, C_1(m_e, M_{H_{1,2}}, M_1) + m_e\, C_0(m_e, M_{H_{1,2}}, M_1)\right) \\
&\qquad + x_2\, d_{21,12}\ \left(\pm\not{p}\, C_1(m_e, M_{H_{2,1}}, M_2) + m_e\, C_0(m_e, M_{H_{2,1}}, M_2)\right)\Big] \\
&- \frac{y^{123}}{\sqrt{2}\,e^2}\Big[- x_{H_1}\, d_{11,12}\ \left(\not{p}\, C_1(m_H, M_{1,2}, M_{H_1}) \mp m_H\, C_0(m_H, M_{1,2}, M_{H_1})\right) \\
&\qquad + x_{H_2}\, d_{21}\ \left(\pm\not{p}\, C_1(m_H, M_{2,1}, M_{H_2}) + m_H\, C_0(m_H, M_{2,1}, M_{H_2})\right)\Big] \\
&\mp \frac{1}{2}\left(\frac{y^{123}}{e}\right)^2\Bigg[x_{H_1}\Bigg[\left(\pm p^2 + \not{p}(m_H \pm m_e)\right) C_1(m_H, m_e, M_{H_1}) \\
&\qquad\qquad \pm \frac{1}{M_{H_1}^2 - m_H^2}\left(M_{H_1}^2\, B_0(M_{H_1}, m_e) - m_H^2\, B_0(m_H, m_e)\right) \\
&\qquad\qquad + \left(\not{p} m_H + m_e m_H\right)\, C_0(m_H, m_e, M_{H_1})\Bigg] \\
&\qquad + x_{H_2}\Bigg[\left(\pm p^2 - \not{p}\,(m_H \pm m_e)\right) C_1(m_H, m_e, M_{H_2}) \\
&\qquad\qquad \pm \frac{1}{M_{H_2}^2 - m_H^2}\left(M_{H_2}^2\, B_0(M_{H_2}, m_e) - m_H^2\, B_0(m_H, m_e)\right) \\
&\qquad\qquad - \left(\not{p} m_H - m_e m_H\right)\, C_0(m_H, m_e, M_{H_2})\Bigg]\Bigg] \\
&\mp \frac{1}{2}\left(\frac{y^{123}}{e}\right)^2\Bigg[x_1\Bigg[\pm\left(p^2 + \not{p}\,(m_H \pm m_e)\right) C_1(m_e, m_H, M_1) \\
&\qquad\qquad \pm \frac{1}{M_1^2 - m_e^2}\left(M_1^2\, B_0(M_1, m_H) - m_e^2\, B_0(m_e, m_H)\right) \\
&\qquad\qquad + \left(\not{p} m_e + m_e m_H\right)\, C_0(m_e, m_H, M_1)\Bigg] \\
&\qquad + x_2\Bigg[\pm\left(p^2 - \not{p}\,(m_H \pm m_e)\right) C_1(m_e, m_H, M_2) \\
&\qquad\qquad \pm \frac{1}{M_2^2 - m_e^2}\left(M_2^2\, B_0(M_2, m_H) - m_e^2\, B_0(m_e, m_H)\right)
\end{aligned}
$$

$$
\begin{aligned}
&\qquad\qquad + \left(m_e m_H - \not{p} m_e\right)\, C_0(m_e, m_H, M_2)\Big]\Big] \\
&- \Bigg[x_1\Bigg[\mp \left(p^2 - \not{p}\,(m_{\tilde\gamma} \mp m_e)\right) C_1(m_e, m_{\tilde\gamma}, M_1) \\
&\qquad \mp \frac{1}{M_1^2 - m_e^2}\left(M_1^2\, B_0(M_1, m_{\tilde\gamma}) - m_e^2\, B_0(m_e, m_{\tilde\gamma})\right) \\
&\qquad + \left(-\not{p} m_e + m_e m_{\tilde\gamma}\right)\, C_0(m_e, m_{\tilde\gamma}, M_1)\Bigg] \\
&\quad - x_2\Bigg[\mp \left(p^2 + \not{p}\,(m_{\tilde\gamma} \mp m_e)\right) C_1(m_e, m_{\tilde\gamma}, M_2) \\
&\qquad\quad \mp \frac{1}{M_2^2 - m_e^2}\left(M_2^2\, B_0(M_2, m_{\tilde\gamma}) - m_e^2\, B_0(m_e, m_{\tilde\gamma})\right) \\
&\qquad\quad + \left(m_e m_{\tilde\gamma} + \not{p} m_e\right)\, C_0(m_e, m_{\tilde\gamma}, M_2)]\Bigg] \\
&+ x_{1,2}\Bigg[\frac{1}{2}\left(B_0(M_{1,2}, 0) + B_0(m_e, 0) + \left(m_e^2 + M_{1,2}^2\right)\, C_0(m_e, 0, M_{1,2})\right) \\
&\qquad + \not{p} m_e \left(C_0(m_e, 0, M_{1,2}) - C_1(m_e, 0, M_{1,2})\right) \\
&\qquad - p^2 C_1(m_e, 0, M_{1,2})\Bigg] \\
&+ \frac{m_{\tilde\gamma}}{\sqrt{2}\, f_0}\left[\mp \not{p} B_1(m_{\tilde\gamma}, M_{2,1}) + m_{\tilde\gamma} B_0(m_{\tilde\gamma}, M_{2,1})\right] \\
&+ \frac{3\tilde{A}_{123}}{e^2}\left[d_{21}\, B_0(M_{2,1}, M_{H_2}) + \{-d_{11}, d_{12}\}\, B_0(M_{1,2}, M_{H_1})\right] \\
&\mp \frac{3\tilde{A}_{123}\, y^{123}}{\sqrt{2}\, e^2}\big[- \not{p}\, B_1(m_H, M_{1,2}) \pm m_H\, B_0(m_H, M_{1,2}) \\
&\qquad\qquad \mp \not{p} B_1(m_H, M_{2,1}) \mp m_H\, B_0(m_H, M_{2,1})\big] \\
&+ \frac{3\tilde{A}_{123}\, y^{123}}{\sqrt{2}\, e^2}\big[\mp (\not{p} + m_e)\, B_0(M_{H_1}, m_e) \mp \not{p}\, B_1(M_{H_1}, m_e) \\
&\qquad\qquad \mp (\not{p} - m_e)\, B_0(M_{H_2}, m_e) \mp \not{p}\, B_1(M_{H_2}, m_e)\big] \\
&+ \frac{m_{\tilde\gamma}}{\sqrt{2}\, f_0}\big[\mp \frac{\not{p}}{m_{\tilde\gamma}^2}\left[\left(p^2 - m_e^2 + m_{\tilde\gamma}^2\right)\, B_1(m_{\tilde\gamma}, m_e) - (p^2 - m_e^2)\, B_1(0, m_e)\right] \\
&\qquad + 3m_{\tilde\gamma}\, \left(p^2 - \not{p} m_e\right) C_1(m_{\tilde\gamma} m_e, 0) \\
&\qquad + \left(\pm\not{p} + 3\,(m_{\tilde\gamma} \mp m_e)\right) B_0(m_{\tilde\gamma}, m_e) \\
&\qquad \pm 3\not{p}\, B_1(m_{\tilde\gamma}, m_e) + \theta_{\mathrm{DREG}}\left(-2(m_{\tilde\gamma} \mp m_e) \mp \not{p}\right)\big]
\end{aligned}
$$

$$
\pm \frac{m_{\tilde\gamma}}{\sqrt{2}\, f_0}\bigg[-\frac{\not{p}}{m_{\tilde\gamma}^2}\left[\left(p^2 - M_{1,2}^2 + m_{\tilde\gamma}^2\right)\, B_1(m_{\tilde\gamma}, M_{1,2}) - (p^2 - M_{1,2}^2)\, B_1(0, M_{1,2})\right]
- 2\not{p}\, B_0(m_{\tilde\gamma}, M_{1,2})\bigg]\bigg\}. \tag{D.17}
$$

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